MA386 Project 1 - Laguerre's Method

Laguerre's method is "super fast" way to find the roots of a polynomial. When you're done with this project, you'll know all about it.

Figure 1: Here is a picture of a guy named Laguerre (Mr. Natural USA, 2004). I thought Laguerre would look older; I'm not sure he has much to do with this method.

- Give a brief overview of the method, including the any assumptions behind it. Also, please find me a picture of the real Laguerre.
- Write a `MATLAB` function called `laguerre.m`, whose inputs are

  1. The coefficients of a polynomial, stored in a vector `p`.
  2. A starting value `x0`.
  3. A tolerance `TOL`.
  4. A maximum number of iterations `Nmax`.

  The output will be an approximate root of the polynomial obtained using Laguerre's method. Include a printout of your code in an appendix with the final project. NOTE: Since we will be dealing with polynomials, you might need the `MATLAB` commands `polyval` and `polyder`. Remember that the `help` command is..., well... helpful.

  Make sure to test your code to make sure it's working correctly. For example, the polynomial $P(x) = x^3 - 8x^2 + 17x - 10$ has roots at $x = 1, 2$, and 5. See if your code can find one of them.
- Let's see how Laguerre's method stacks up against Newton's method. Use each method to factor the polynomials below by finding all their roots. As you find each root, factor it out of the polynomial using synthetic division[1], then find the next root using the deflated polynomial. You may use any initial guesses you like, but you must use a tolerance of at least 10^{-8}. Keep track of the number of iterations required to find each root.

  1. $P_1(x) = x^3 - 2.82x^2 - 49.8919x + 195.153408$
  2. $P_2(x) = x^5 - 105x^4 + 2809x^3 - 2809x^2 + 2808x - 2704$
- Discuss the outcome of your experiments. Include any notable differences between Laguerre's method and Newton's method. Make sure your discussion includes the order of convergence of the two methods, and provide numerical evidence backing up your claims.
- Include all of the above in a typed, professionally written report. If you need to include any by-hand work, put it in an appendix. NOTE: I can also read Wikipedia; try to put any ideas you find into your own words, and be sure to document your sources. If you have *any* questions at all, let me know immediately. Good luck!

[1] You can do this by hand if you wish, but... 10 pt. BONUS - write a MATLAB function whose inputs are $P(x)$ and its root p that will output the polynomial Q, where $P(x) = (x - p)Q(x)$ (i.e. implement Horner's Method). I'd be glad to help you with this if you get stuck.