

# WPR II Topics

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## 1. Functions of Several Variables & Estimations

- What are the three rules for finding the domains of these functions?
- Understand how to work with these functions whether given as  $f(x, y)$ , in a table of values, or in a contour plot (i.e. level curves).
- Understand how to estimate net volumes using functions of two variables, given in all of the 3 types above. (How: find volumes of "bricks". The heights of the bricks are given by the function value at some sample point in your sub-rectangle.) Refine your estimates in Mathematica.
- Understand how to over/under-estimate the net volume.

## 2. Double Integration (**Big** topic!!)

- Understand how we defined the double integral symbol (Lesson 23) (i.e. as the limit of the estimation process in the previous section).
- What does the symbol  $\iint_R f(x, y) dA$  mean geometrically?
- Know how to compute iterated integrals *by hand* by moving from the "inside-out." (This goes for all types of regions.)
- Fubini's Theorem (i.e. how to switch the order of integration when integrating over a rectangle).
- Integration over general regions, including how to switch the order of integration. (**Always** draw a picture of the base region!)
- Know how to use double integrals to compute the mass and center of mass of a thin plate (lamina). Remember, it takes three double integrals to compute the center of mass.
- Integration over polar regions. Recognize when you may need to use this and how to execute the switch from rectangular to polar integrals.
- Physical quantities and double integration: remember that if you double integrate any type of density function (any mass per area) over a region, you are computing the total amount of mass in that region. (e.g. total population for population density, total charge for charge density, total mass for mass density...)

## 3. Parametric Equations, Vector Functions, Arc Length, and Motion in Space

- Be comfortable with parametric equations and vector functions given in 3 different forms:  $\mathbf{i}, \mathbf{j}, \mathbf{k}$  notation, bracket notation, or as  $x = , y = , z = .$
- Understand what a parametric curve is and how to sketch one (by hand and with Mathematica). Know how to find where it starts, stops, what direction it travels, etc.
- Know how to differentiate and integrate vector functions. This includes solving for constants  $C$  in indefinite integrals.
- Know how to compute the length of a parametric curve, or the total distance traveled by a particle. (i.e. arc length)

- Know the relationships between all of the following: position, velocity, speed, and acceleration. Also, know how to find one given any of the others.
- Understand the derivation for the equations of projectile motion in 2 dimensions.
- Be able to solve projectile motion problems like the ones we did in class and in the do problems. You may have to find any of the following: angle, initial position, initial speed, time, where it lands, ... (and more!)

#### 4. Mathematica: Things we discussed

- Double integrals in Mathematica (including `NIntegrate` if necessary)
- Plotting:
  - Functions of two variables
  - Parametric curves in  $2D$
  - Parametric curves in  $3D$  (space curves)
- Vector functions in Mathematica (and how to integrate them)
- Arc length
- Solve command (primarily for projectile problems)