MA205 - Integral Calculus
Homework Assignment 4
Due In Class Monday, October 22

In working the problems below, some will utilize Mathematica, and some will require by-hand calculations. In problems where Mathematica is used to perform integration, you **must** show the complete set up of the integral by hand to receive full credit.

1. Change the following integral to polar coordinates, then evaluate it by hand. Be sure to show all your work.
$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}} e^{-(x^2+y^2)}\,dx\,dy$$

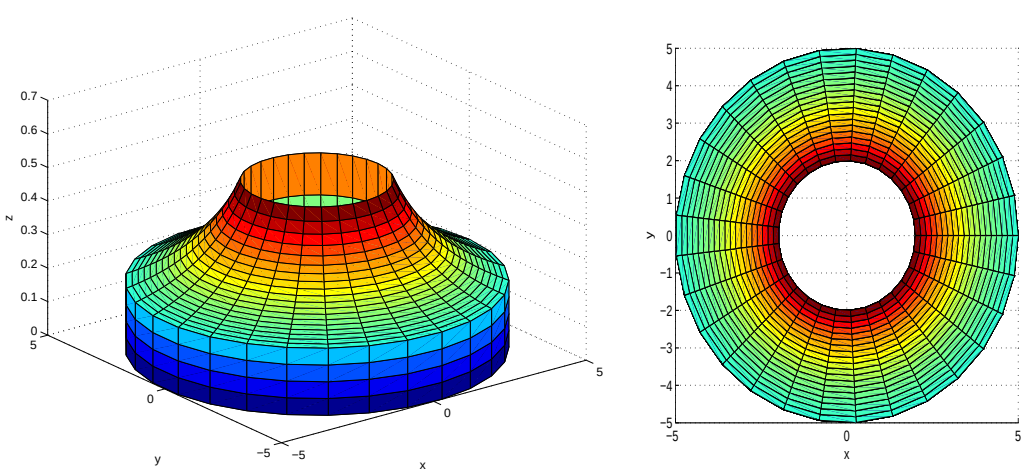
2. Describe the path of the particle with position function below. You must be very thorough to receive full credit. Include a plot (can be hand drawn) in your analysis.
$$\mathbf{r}(t) = \langle (\sin(t))^3,\ \sin(t)\rangle,\ \ 0 \le t \le 13.$$

3. A Hot Pocket travels through space with a velocity of $\mathbf{v}(t) = t^2\mathbf{i} + \sin(4t)\mathbf{k}$. If it started at the point $(2, 3, -1)$, what is its position function? What is the acceleration function?

4. A slightly stretched Slinky can be modeled by the following parametric equations, where x, y, and z are measured in feet and $0 \le t \le 22$.
$$\begin{aligned} x(t) &= \sin(2t) + 0.1t \\ y(t) &= \cos(2t) + 0.1t \\ z(t) &= .04t \end{aligned}$$
How long is it? Mathematica may require the `NIntegrate` command.

5. A Gummi Bear flies through the asteroid belt with an acceleration given by $\mathbf{a}(t) = \langle 1 + t^2, 5, \sqrt{t}\rangle$. If it has an initial velocity of $\mathbf{v}(1) = \langle 0, -2, 1\rangle$, what is its speed when $t = 2$?

6. Find the volume of the following geometric object (on the left; the right picture is a top view). It may not look like it, but the answer is really nice. The top of the object is the graphed surface of the function $f(x, y) = 1/\sqrt{x^2 + y^2}$.



7. BONUS (2 pts) Continuing from number 6, what is the volume if we keep the same function describing the top of the solid, but the object has no hole in the middle? Will the volume be be infinite or finite? Note: the function "goes to infinity" at the origin.