MA205 - Integral Calculus
Homework Assignment 5
Due to Thayer 216 by 1600, Tuesday, November 13

On the problems below that do not specify by hand calculations, Mathematica is encouraged. In each problem, you must make clear which method you use to solve the differential equation, and include any relevant Mathematica commands you use. Attach a printout if appropriate. Show all work to receive full credit.

1. Using separation of variables, solve the initial value problem

$$y' = y^2 \sin(x),\ y(\pi) = 2$$

by hand. Then, use your solution to determine $y(0)$. (You should only use Mathematica to *check* your work on this problem!)

2. Using separation of variables, solve the initial value problem

$$\frac{dy}{dt} = t^2 y,\ y(0) = 2$$

by hand. (You should only use Mathematica to *check* your answer.)

3. A bacteria culture starts with 120 bacteria and grows with constant relative growth rate. After 8 hours the count is 75,000.

   (a) Find an expression for the number of bacteria after t hours.

   (b) When will the population reach $200,000$?

4. The Pacific halibut fishery has been modeled by the differential equation

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{K}\right)$$

where $y(t)$ is the biomass (the total mass of the members of the population) in kilograms at time t (measured in years). The carrying capacity is estimated to be $K = 8 \times 10^7$ kg and $k = 0.71$ per year. If $y(0) = 2 \times 10^7$ kg, find the biomass a year later.

5. A tank contains 1000L of pure water. Brine that contains 0.05kg of salt per liter of water enters the tank at a rate of 5L/min. Brine that contains 0.04kg of salt per liter of water enters that tank at a rate of 10L/min. The solution is kept well mixed and drains from the tank at a rate of 15L/min.

   (a) Set up a differential equation that models the above system, remembering to name your variables and include initial conditions.

   (b) Using Euler's method with a step size of 0.5, calculate how much salt is in the tank after 10 minutes.

   (c) Repeat the above calculation with a step size of 0.25.

   (d) Now use the `DSolve` command and write down an exact solution to the IVP. According to this solution, how much salt is left in the tank after 10 minutes?

6. Work and turn in Problem Solving Problem #3 (a-b) from Lesson 47. This is on page 242 of your student guide.