

# MA205 - Integral Calculus

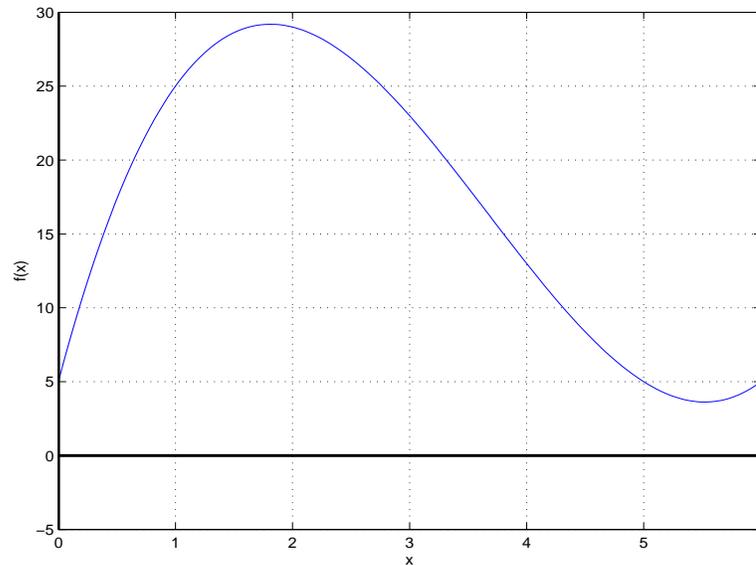
## Lesson 6: The Definite Integral

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1. The function  $f(x)$  is graphed below. Sketch the graphical representation of the definite integrals on the graph provided.

(a)  $\int_1^3 f(x)dx$

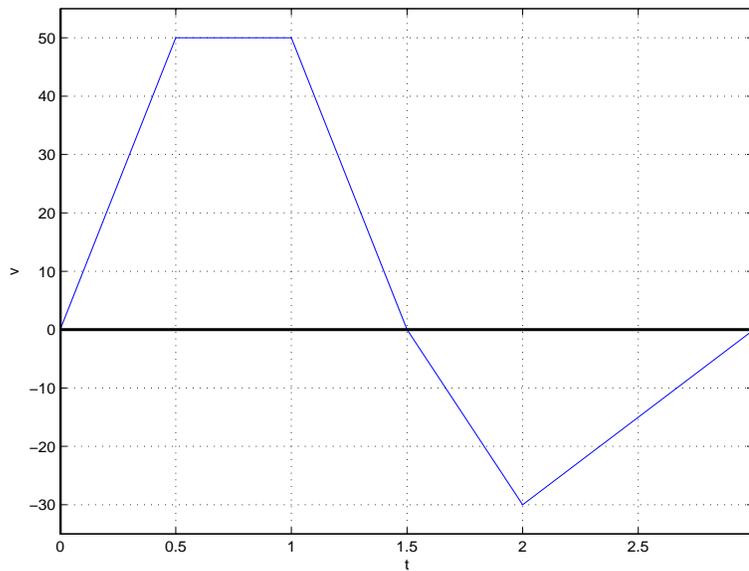
(b)  $\int_3^4 f(x)dx$



- (a) Given  $\int_1^3 f(x)dx = 54$  and  $\int_3^4 f(x)dx = 18$ , what is  $\int_1^4 f(x)dx$ ? Can you see this on the graph?

- (b) Given  $\int_1^3 f(x)dx = 54$  and  $\int_1^6 f(x)dx = 85$ , what is  $\int_1^6 f(x)dx - \int_1^3 f(x)dx$ ? How is it related to  $\int_3^6 f(x)dx$ ? Can you see this on the graph?

2. Below is a graph representing the velocity of a car, given by the function  $v(t)$  (in meters per hour), versus time (in hours). Find the following quantities. What do they mean to the driver?



(a)  $\int_0^{1.5} v(t) dt$

(b)  $\int_{1.5}^3 v(t) dt$

(c) Use the previous values to determine  $\int_0^3 v(t) dt$

(d) What does  $\int_0^3 |v(t)| dt$  represent physically?

3. Do numbers 47-49 on page 392. The properties on page 388 may be helpful.

4. UNITS!!! If we integrate velocity (the rate of change of position), we obtain a displacement. If we integrate a rate of change, we obtain a new quantity. Let the units be your guide...

(a) If  $P'(t)$  is the rate of growth of a population  $P(t)$  at time  $t$  (in years), what does  $\int_0^1 P'(t) dt$  represent?

(b) If water is pumped out of a lake at a rate  $r(t)$  gallons per minute, what does  $\int_0^1 r(t) dt$  represent?  
How about  $\int_3^5 r(t) dt$ ?

(c) The current in a wire is defined as the derivative of the charge:  $I(t) = Q'(t)$ . What does  $\int_a^b I(t) dt$  represent?