MA205 - Integral Calculus
Lesson 22: Estimations Using Functions

1. In this problem, we will estimate the volume of the solid between the graph of $f(x, y) = e^{-x^2-y^2}$ and the region R given by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

   (a) First estimate using lower right hand sample points with six regions. (Hint: go ahead and put this function into Mathematica to make calculating the heights of your bricks easier.)

   (b) In integral notation, what is the quantity you are estimating?

   (c) Now let's practice using the Mathematica code in your student guide. Start by using Plot3D to visualize what this solid looks like. Now use the Riemann sum commands to estimate the volume of the solid with 100 "bricks".

   (d) How much volume do you think is in between the graph of $f(x, y)$ and the *entire* xy-plane? Try to guess the answer by looking at larger and larger regions R. The answer should surprise you, especially since when you consider the whole xy plane, you are considering an infinite region!

2. What is the volume under a sphere of radius 4 centered at the origin and over the square R given by $-1 \leq x \leq 1$, $-1 \leq y \leq 1$? Remember, the equation of a sphere is $x^2 + y^2 + z^2 = r^2$.

   (a) What is the integral notation for the quantity your are computing?

   (b) Estimate the volume by hand using 4 sub-regions.

   (c) Now use Mathematica to refine your estimate and try to get an exact answer.

3. Finish any unworked problems in your student guide from Lessons 21 and 22.