

## MA205 - Integral Calculus

### Lesson 30: Polar Regions II

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- Remember, when working in polar coordinates the “ $dA$ ” in the integral  $\int \int_R f(x, y) dA$  is equal to  $r dr d\theta$ ! (or  $r d\theta dr$ , depending on the order you choose).

#### Problems

1. Evaluate  $\int \int_D xy dA$  where  $D$  is the disk centered at the origin and having radius 3.

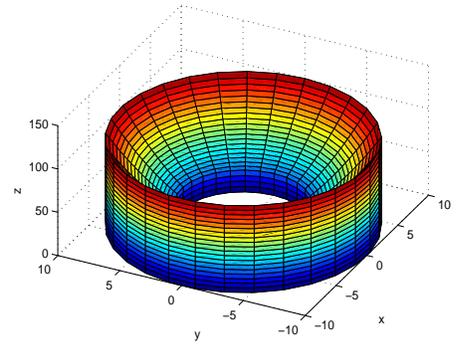
2. Let's use polar coordinates to evaluate the following integral:  $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{x^2+y^2} dy dx$ .

(a) First sketch the region of integration. Mathematica may help.

(b) Now describe the region in polar coordinates. This example should be a polar rectangle.

(c) Next, re-write the integral in terms of polar coordinates, including the limits of integration. Then integrate!

3. What is the volume of the region between the circle of radius 5 and the circle of radius 10 under the function  $f(x, y) = x^2 + y^2$ ?



4. Use polar coordinates to combine the following integrals into one. Hint: draw each region of integration.

$$\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} f(x, y) dy dx + \int_0^{\sqrt{3}} \int_1^{\sqrt{4-y^2}} f(x, y) dx dy$$

5. Work the Problems 2 and 5 in your course guide problems on polar regions. Then, if you have time, work any other unfinished problems on polar regions from your course guide.