

# MA383 Foundations in Mathematics

## Lesson 23: Induction Practice Problems

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Prove the following statements by induction.

### 1. Easy

(a) For all  $n \in \mathbb{N}$ ,  $\sum_{i=0}^n 2^i = 2^{n+1} - 1$ .

(b)  $\sum_{i=0}^{n-1} (2i + 1) = n^2$  for all  $n \in \mathbb{N}$ .

(First, be sure you understand why this is the same as proving that the sum of the first  $n$  odd positive integers is  $n^2$ .)

### 2. Medium

(a)  $3^n < n!$  for all integers  $n \geq 7$ .

(b)  $\sum_{i=0}^n 3 \cdot 5^i = \frac{3(5^{n+1} - 1)}{4}$  for all  $n \in \mathbb{N}$ .

(c) For all integers  $n \geq 1$ ,  $3 | (2^{2n} - 1)$ .

(d)  $\sum_{i=1}^{n+1} i2^i = n2^{n+2} + 2$  for all integers  $n \geq 0$ .

### 3. Difficult

(a)  $2^n > n^2$  whenever  $n$  is an integer greater than 4. (Hint: We proved an exercise using induction in class that you can/should use within your proof)

(b)  $n! < n^n$  for all integers  $n \geq 2$ .