MA383 Foundations in Mathematics
Lesson 31: Functions on Sets

Let $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ and define

$$f : \mathbb{Z}_6 \to \mathbb{Z}_6$$

by

$$f(x) = (x^2 + 3) \pmod 6.$$

Define four subsets of $\mathbb{Z}_6$ by

$$A = \{1, 3, 4\}$$
$$B = \{0, 3, 5\}$$
$$C = \{0, 1, 2\}$$
$$D = \{1, 4, 5\}$$

1. Compute $f(x)$ for each $x \in \mathbb{Z}_6$. If you find it helpful, catalog this information into a diagram representing the function.

2. Compute the following sets:

   (a) $A \cap B$

   (b) $C \cap D$

   (c) $A \cup B$

   (d) $C \cup D$

3. Compute the following images (and combinations of images) and make conjectures on the relationships between them, based on your findings:

   (a) $f(A \cap B)$

   (b) $f(A \cup B)$

   (c) $f(A) \cap f(B)$

   (d) $f(A) \cup f(B)$

4. Compute the following preimages (and combinations of preimages) and make conjectures on the relationships between them, based on your findings:

   (a) $f^{-1}(C \cap D)$

   (b) $f^{-1}(C \cup D)$

   (c) $f^{-1}(C) \cap f^{-1}(D)$

   (d) $f^{-1}(C) \cup f^{-1}(D)$