MA104 - Differential Calculus
Homework Assignment 4 - 35 points
Due in class Monday, April 21, 2008

Be sure to show all work to ensure full credit!

1. (4 points) Let $f(x,y) = \dfrac{6x^3y}{2x^4+y^4}$. Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist by finding two paths along which (x,y) approaches $(0,0)$ but $f(x,y)$ approaches two different heights. (Hint: Look at the contour plot of this function to find a clue as to what types of paths will give rise to different function values.)

2. (5 points) Recall the table of Army officer basic pay that we used in class. (This was in Lesson 45, so if you lost it, you will find it posted on my website.) Recall we defined a function $P(r,y)$, where P is monthly pay, r corresponds to the rank Or (i.e. $r=3$ is a captain), and y is years of service.

   (a) Use the table to estimate $P_y(3,8)$.

   (b) Describe the practical interpretation of your previous answer.

   (c) When does a captain's pay cease to increase, even with additional years of service to the Army? Describe this behavior with a mathematical statement involving the function P.

3. (3 points each) For the following functions, find f_{xx}, f_{yy}, and f_{xy} **by hand, showing all of your work.** Only use Mathematica as a means for checking your work.

   (a) $f(x,y) = \sqrt{x}\ln(y)$

   (b) $f(x,y) = \sin(xy) + x^2y^5$

4. (2 points each) Compute the directional derivative of the function **by hand** at the given point P in the direction of the vector $\mathbf{v}$. Remember you need two things to compute a directional derivative: a *unit* vector and the gradient of a function.

   (a) $f(x,y) = 3x^3y - \sin(y)$, $P=(1,1)$, $\mathbf{v} = \langle 3,-2\rangle$.

   (b) $g(x,y) = x^2+y^2$, $P=(1,-2)$, $\mathbf{v} = \langle 2,1\rangle$.

5. (4 points) Work problem 30 on page 921.

6. (3 points) If $f(x,y)$ is a function representing elevation and you are standing at the point $P = (x_0, y_0, f(x_0,y_0))$, what is the significance of the gradient vector at P?

7. (4 points) Let $g(x,y) = \sin(xy) - y^2 + 1$ and $P=(1,2)$. Find the maximum rate of change of g at P and the direction in which it occurs *(see Theorem 15, page 916)*. Complete this problem by hand, but feel free to check your work with Mathematica.

8. (5 points) Let $f(x,y) = e^y(y^2-x^2)$.

   (a) Find all the critical points of f. (Note: Feel free to use Mathematica, but attach a printout if you do, and comment your code to explain what you were doing.)

   (b) Use the second derivative test to classify each critical point as a local max, local min, or saddle point.