MA104 - Differential Calculus
Lesson 2: Limit of a Sequence
Class Problems

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1. Algebraically (i.e. by hand) compute the limits of the following sequences:

   (a) $a_n = \dfrac{n^3}{n+1}$

   (b) $b_k = \dfrac{5^{k+3}}{7^k}$ (Hint: See one of the red boxes on page 681.)

2. Graph the following sequences using Mathematica's `ListPlot` command. Using your plot, determine whether the sequence is convergent or divergent. If the sequence is convergent, use your graph to estimate the value of the limit.

   (a) $a_n = 1 + \left(-\dfrac{2}{e}\right)^n$

   (b) $c_m = \dfrac{m \cos m}{1+m}$

3. Consider the sequence $y_n = \left(1 + \dfrac{1}{n}\right)^n$.

   (a) List one reason why you might expect $\lim\limits_{n\to\infty} y_n$ to equal 1.

   (b) List one reason why you might expect $\lim\limits_{n\to\infty} y_n$ to diverge to ∞.

(c) Now use Mathematica's `Table` command to list the first 50 terms in the sequence. What does the sequence seem to be approaching? Can you write this number *without* using a decimal? (Hint: Yes, you can, it is a very famous number!)

4. Compute $\lim_{k\to\infty} \sqrt{k}\sin(\pi/\sqrt{k})$. (Make your method clear.)

5. Compute $\lim_{n\to\infty} \dfrac{\sqrt{2n^4+18n^2}}{n^2+7n}$ using two different methods.