

MA104 - Differential Calculus

Lesson 3: Limit of a Function I

Group Exercises

Group 1: Carefully read Definition 1 on Page 88 of Stewart. Find a way to put this definition into your own words. Now sketch two functions. First, sketch a function $f(x)$ so that $\lim_{x \rightarrow 3} f(x) = 8$. Then, sketch a function $f(x)$ so that $\lim_{x \rightarrow 3} f(x) \neq 8$. For both, be sure you can explain why the limit definition is either satisfied or violated. Get ready to share your interpretation of the definition and your examples with the class.

Group 2: Carefully read Definition 2 on Page 93 of Stewart. Find a way to put this definition into your own words. Now sketch two functions. First, sketch a function $f(x)$ so that $\lim_{x \rightarrow 1^-} f(x) = 2$. Then, sketch a function $f(x)$ so that $\lim_{x \rightarrow 1^-} f(x) \neq 2$. For both, be sure you can explain why the limit definition is either satisfied or violated. Get ready to share your interpretation of the definition and your examples with the class.

Group 3: Sketch examples of functions $f(x)$ which have the following properties:

1. $\lim_{x \rightarrow 2} f(x) = 4$
2. $\lim_{x \rightarrow 2^-} f(x) = 4$ and $\lim_{x \rightarrow 2^+} f(x) = 2$
3. $\lim_{x \rightarrow 3^-} f(x) = 0$ but $f(3)$ is undefined

Get ready to show (and explain) your sketches to the class.

Class Problems

1. For the function whose graph is shown, state the value of each quantity, if it exists. If it does not exist, explain why.

a. $\lim_{x \rightarrow 0^-} f(x)$

b. $\lim_{x \rightarrow 0^+} f(x)$

c. $\lim_{x \rightarrow 0} f(x)$

d. $f(1)$

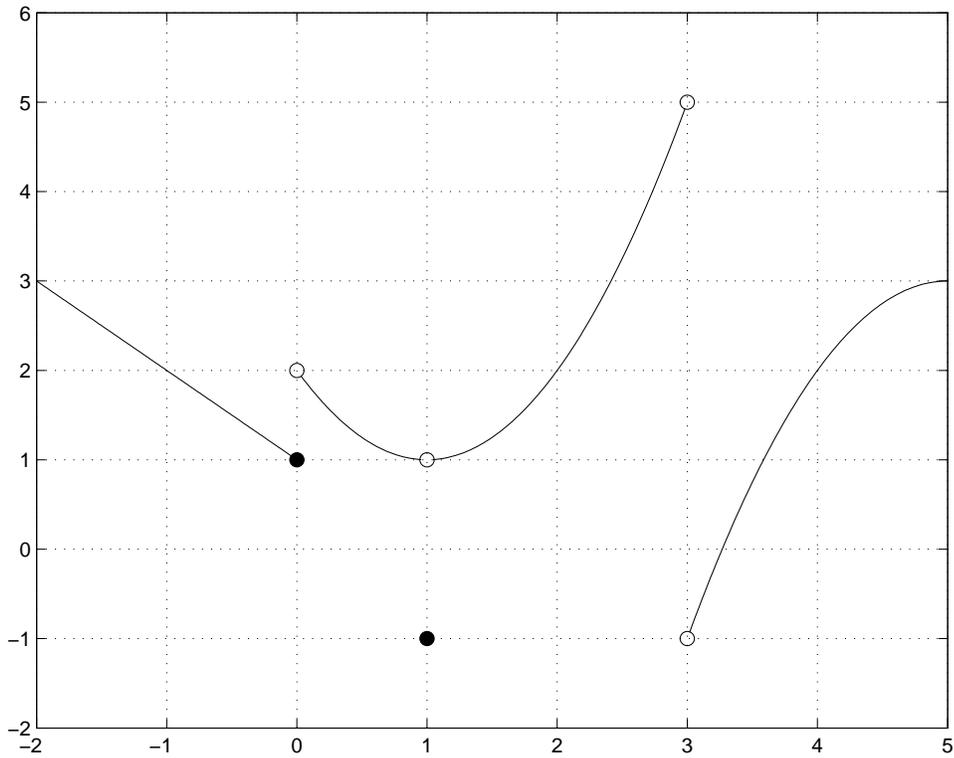
e. $\lim_{x \rightarrow 1} f(x)$

f. $\lim_{x \rightarrow 3} f(x)$

g. $f(3)$

h. $\lim_{x \rightarrow 3^-} f(x)$

i. $\lim_{x \rightarrow 3^+} f(x)$



2. Consider the function $f(x) = \frac{\cos(x) - 1}{x}$. Notice that $f(0)$ does not exist. However, we can still gain information about $\lim_{x \rightarrow 0} f(x)$ by filling out a table of values. Fill in the table, and then use the values you find to guess the limit. Plot the function in Mathematica to verify that your limit is correct.

x	$f(x)$
.2	
.1	
.01	
.001	
	Undefined
-.001	
-.01	
-.1	
-.2	

3. For the function below, find the left-hand and right-hand limits of $f(x)$ as x approaches 0. Does $\lim_{x \rightarrow 0} f(x)$ exist? If not, explain why. Then, graph the function in Mathematica to verify that your answer is correct.

$$f(x) = \frac{1}{1 + e^{1/x}}$$

x	$f(x)$
.2	
.1	
.01	
.001	
	Undefined
-.001	
-.01	
-.1	
-.2	