MA104 - Differential Calculus
Lesson 4: Limit of a Function II

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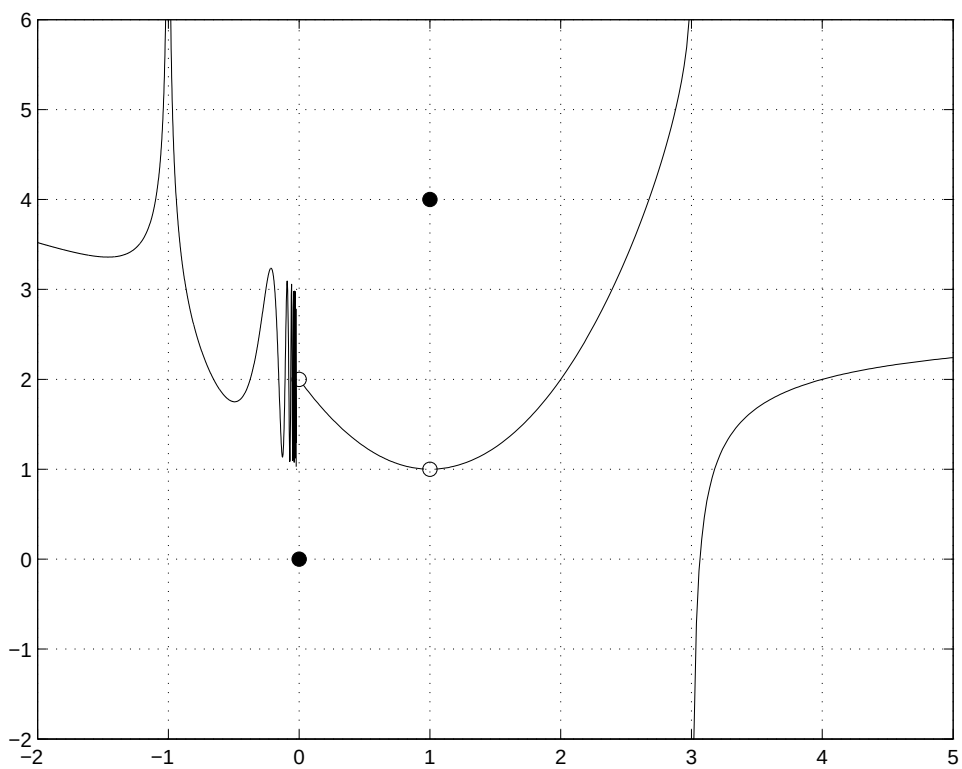
1. For the function whose graph is shown, state the value of each quantity, if it exists. If it does not exist, explain why.

a. $\lim_{x \to 0^-} f(x)$	b. $\lim_{x \to 0^+} f(x)$	c. $f(0)$
d. $f(-1)$	e. $\lim_{x \to -1} f(x)$	f. $\lim_{x \to 3} f(x)$
g. $f(3)$	h. $\lim_{x \to 3^-} f(x)$	i. $\lim_{x \to 3^+} f(x)$
j. $f(1)$	k. $\lim_{x \to 1} f(x)$	



2. Now let's get used to some Mathematica, at the same time as practicing our limit skills. Let

$$f(x) = \frac{5x}{x-2} \qquad g(x) = \frac{x^2-x}{x} \qquad h(x) = \ln(x-1).$$

(a) First, define each of these functions in Mathematica. To define a function in Mathematica, you must use an *extra underscore* in your definition. For example, to define $s(t) = t^2$, you would type `s[t_]=t^2` . Then, the function is saved in Mathematica, allowing you to compute $s(4)$ by typing `s[4]`, for example. (Note, to type ln in Mathematica, use `Log`.)

(b) Investigate $\lim_{x\to 2^+} f(x)$ by calculating $f(2.1)$, $f(2.01)$, $f(2.001)$, and $f(2.0001)$. What is your guess for the limit? Now, use the `Plot` command in Mathematica to plot the function and check your guess. Sketch the function here as well.

(c) Plot the function $g(x)$ in Mathematica. On your paper, sketch what Mathematica gives you. Now think: what x value is *not in the domain* of $g(x)$? Did Mathmematica show this in the plot? Now alter your sketch to include the information you learned about this function. Then compute $\lim_{x\to 0} g(x)$. How can you see this limit algebraically? (i.e. just by manipulating the function g?)

(d) Compute $\lim_{x\to 1^+} h(x)$, $\lim_{x\to 1^-} h(x)$, and $\lim_{x\to 1} h(x)$. Use a table of values (numerical method) or a graph to help you.