

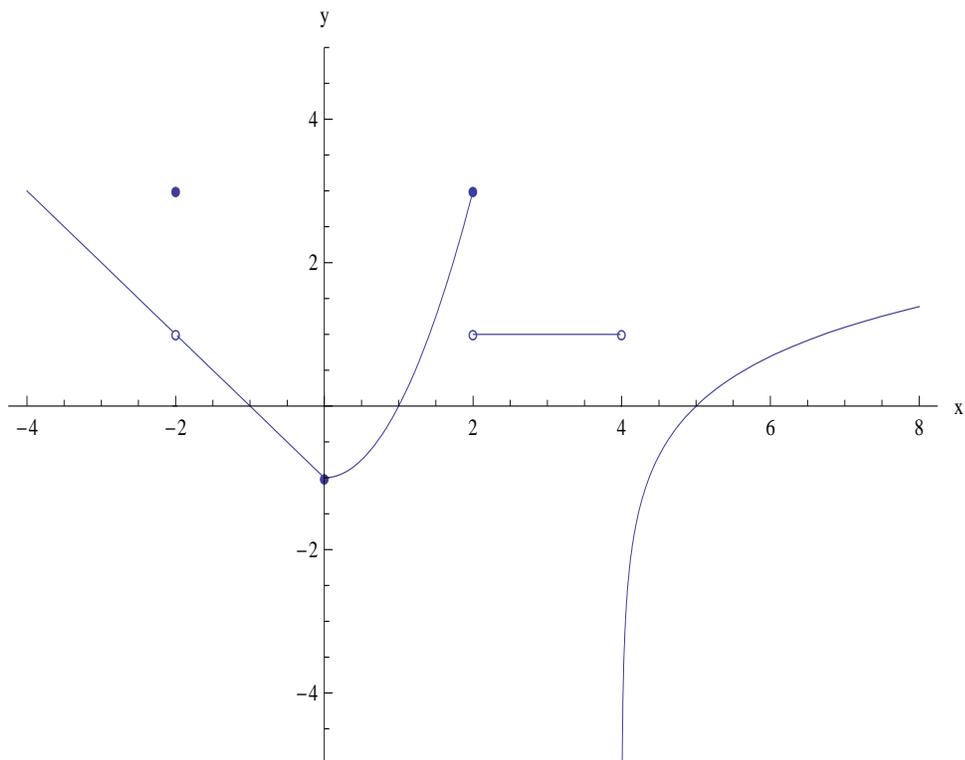
**MA104 - Differential Calculus**  
**Lesson 5: Continuity**  
**Class Problems**

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1. Evaluate  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3}$  algebraically (i.e. by hand). Then, use it to determine if the following function is continuous at  $x = 3$ :

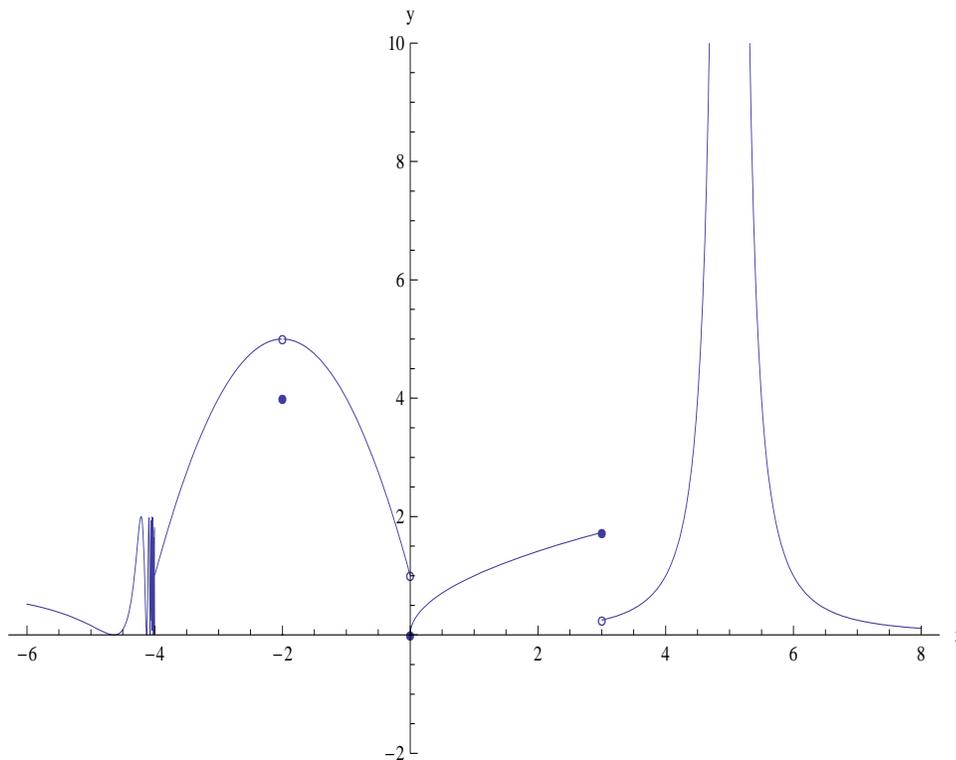
$$f(x) = \begin{cases} \frac{x-3}{x^2-2x-3} & \text{if } x \neq 3 \\ 0.5 & \text{if } x = 3 \end{cases}$$

2. The graph of a function  $f(x)$  is given below.
- (a) Find each value at which  $f$  is discontinuous. State which rule is violated at each point.
- (b) What are the *intervals* on which  $f$  is continuous?



3. The graph of a function  $g(x)$  is given below.

- (a) Find each value at which  $g$  is discontinuous. State which rule is violated at each point.  
 (b) What are the *intervals* on which  $g$  is continuous?



4. Do you think all piecewise functions are discontinuous at the border points? Investigate the piecewise function

$$h(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \ln(x) + 1 & \text{if } 1 \leq x < e \\ x & \text{if } x \geq e \end{cases}$$

by plotting it either by hand or in Mathematica. (Type `??Piecewise` for an example of how to do this.) Then, determine whether  $h(x)$  is continuous at  $x = 1$  and  $x = e$ . Give clear reasons for your answers.