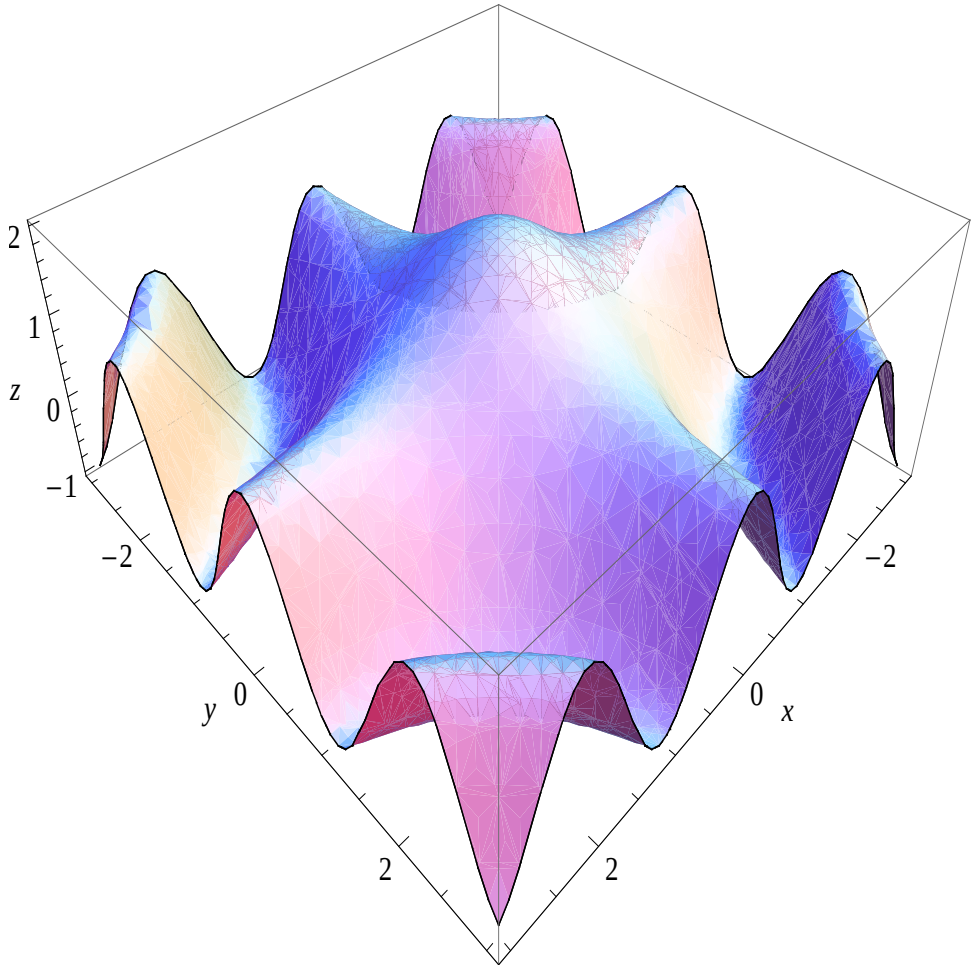
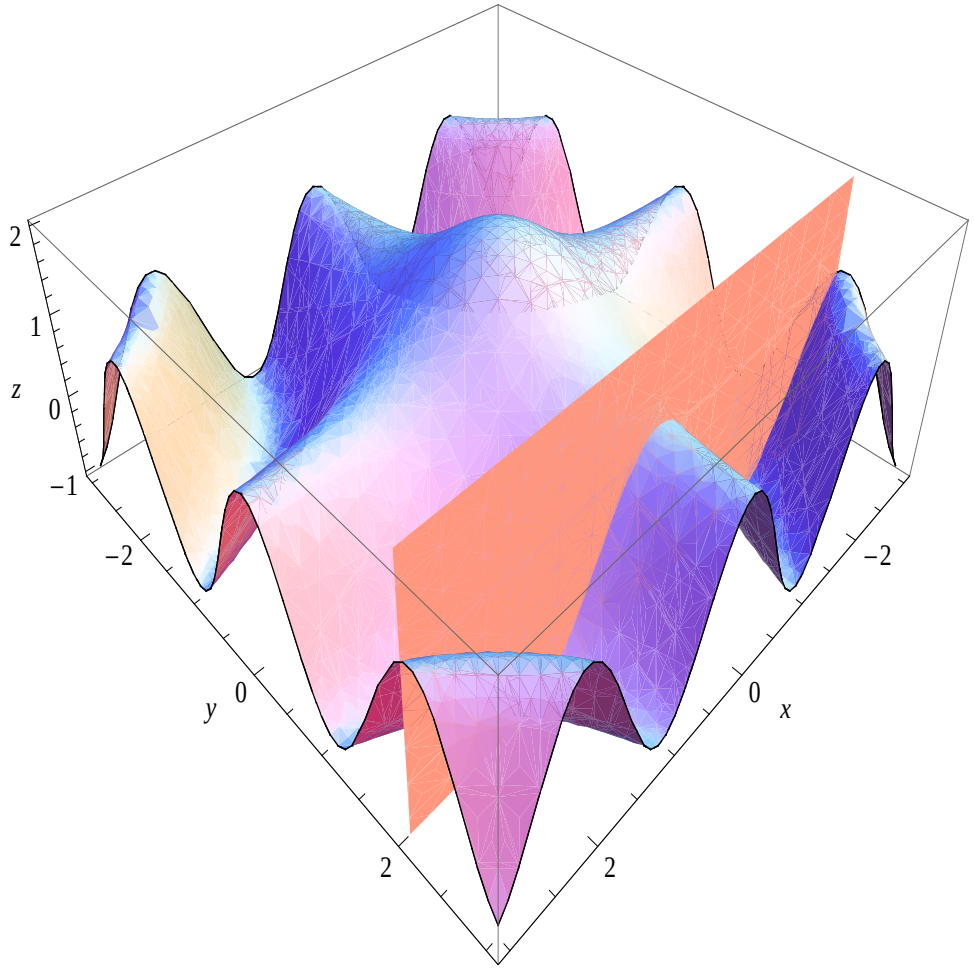
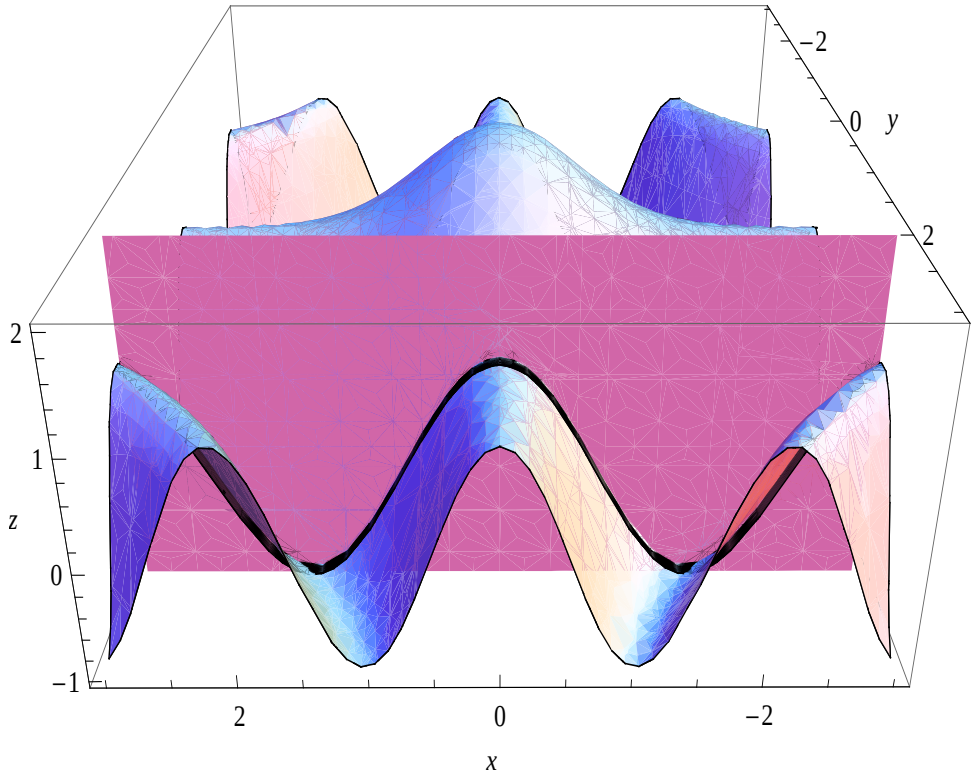


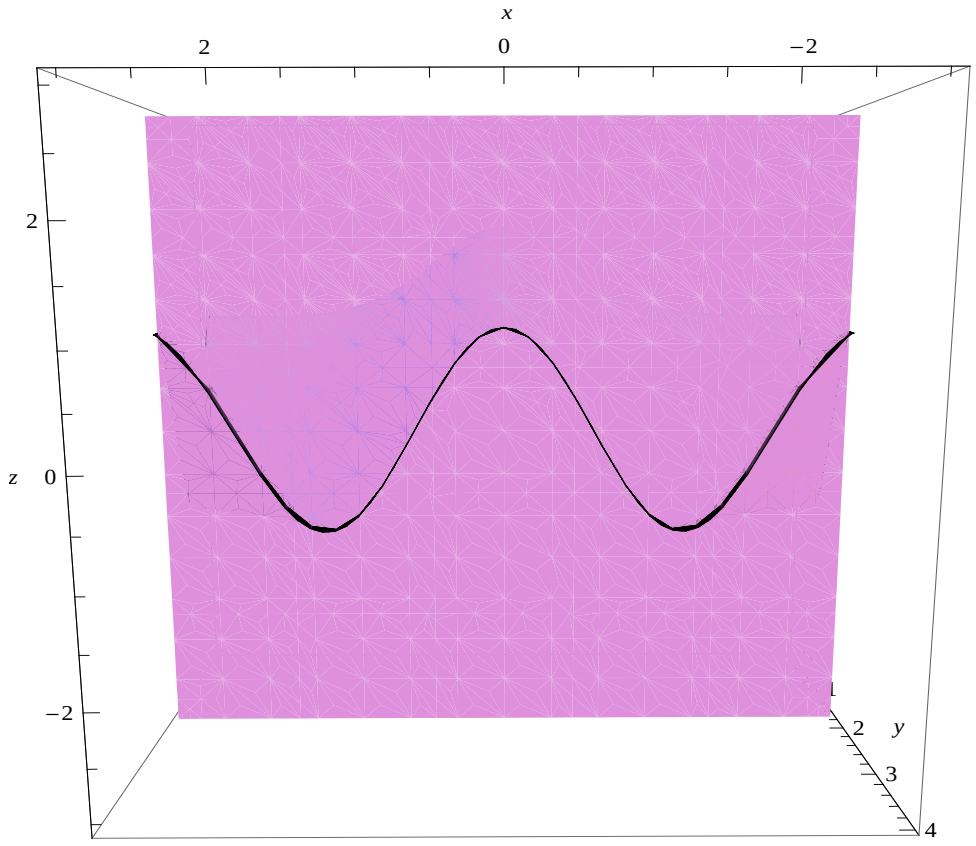
$$z = f(x, y)$$



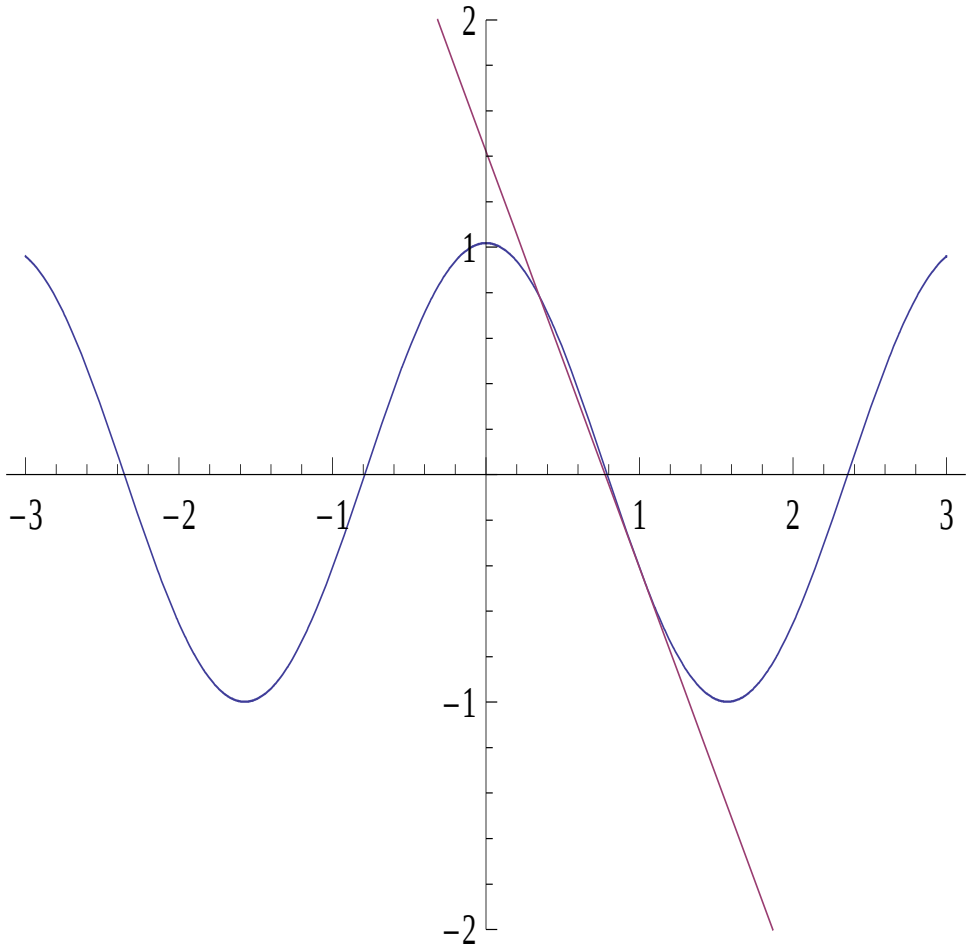
$z = f(x, y)$ and the plane $y = 2$



A side view of the intersection of the surface with the plane.
In black you see $f(x, 2)$.



$f(x, 2)$



$f(x, 2)$ together with the tangent line to this curve at $x = 1$.
i.e. $f_x(1, 2)$ is the slope of this tangent line.