MA104 - Differential Calculus
Lesson 48: The Gradient

1. For the following functions, compute ∇f by hand:

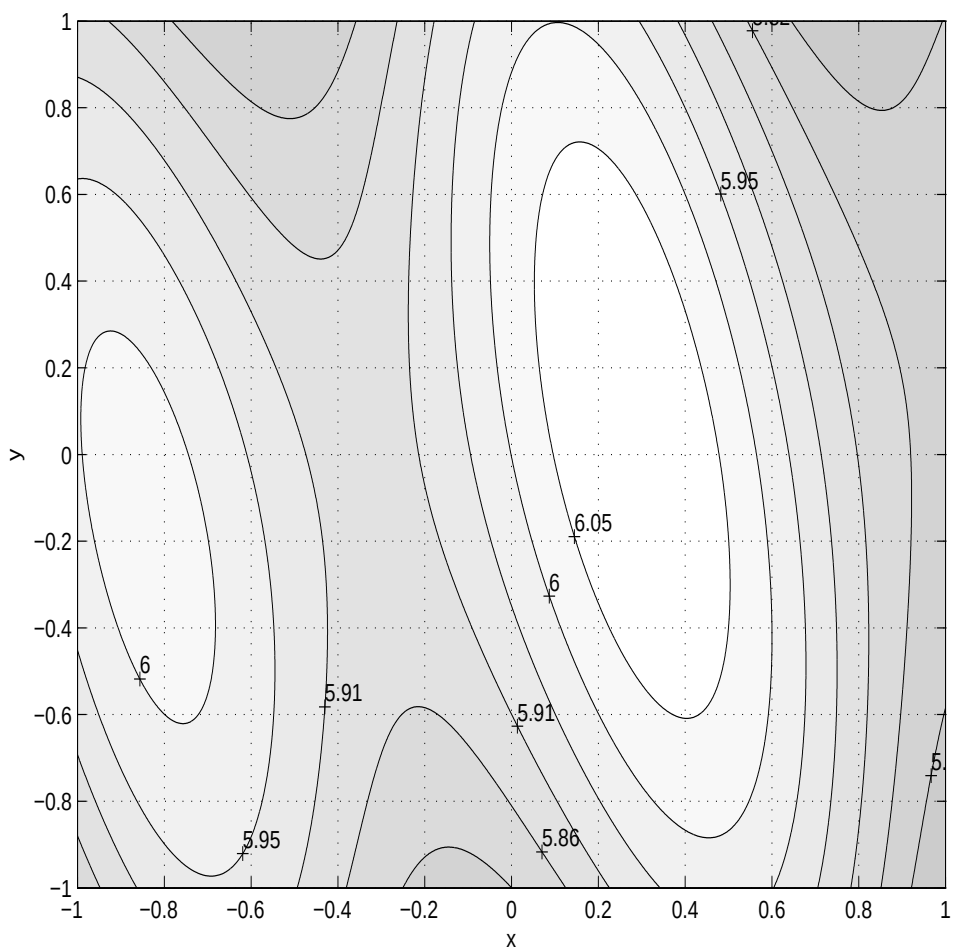
   (a) $f(x, y) = \ln(x^2 + y^2) + x\sqrt{y}$

   (b) $f(x, y) = \sin(x^2 + 3y) + e^{-x^2 y}$

2. Below is a contour plot of the function

$$f(x, y) = 6 + \frac{\sin(5x + y)}{10} - \frac{x^2}{10} - \frac{y^2}{10}.$$

   Calculate and plot the gradient of the function at the points $(-0.8, -0.6)$, $(-0.6, -0.9)$ and $(0, 0)$. What do they all have in common?



3. Use the gradient to find the maximum rate of change of the function $f(x, y) = y^3/x + \sin(2x)$ at the point $(\pi, 1)$ and the direction in which it occurs. (Do this by hand.)

4. (Use Mathematica for this one!) We're going to use the gradient to calculate the path of a hiker traveling up a mountain. The elevation of the terrain is given by the following function (presented "Mathematica Style"):

```
f[x_,y_]= E^(-.5 * (x^2 + y^2))*(1 + x + y^2)*Cos[Sin[x + y]].
```

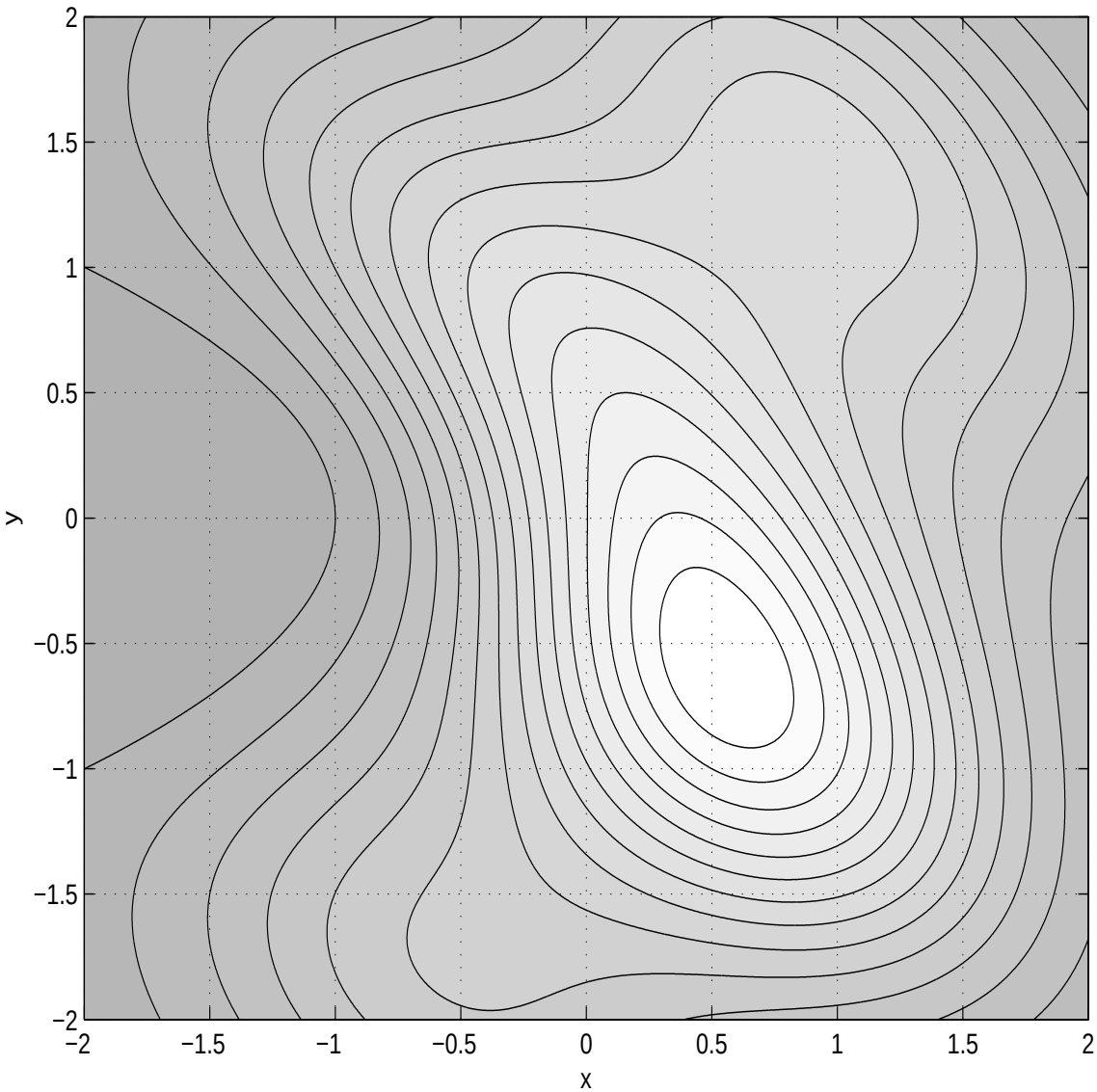
A contour plot of this function is provided for you on the next page. Here is the hiker's plan for getting to the top:

- Starting at the point $(-0.8, -0.6)$, she will travel 0.3 units (as the crow flies) in the direction of **greatest ascent** and then stop.
- Once at her new point, she will "recalibrate" by finding the new direction of **greatest ascent** and then travel 0.3 units in that direction.
- She repeats this until she gets to the top.

Your task is to calculate each point she stops at and plot her path on the contour plot as you go.

```
f[x_,y_]= E^(-.5 * (x^2 + y^2))*(1 + x + y^2)*Cos[Sin[x + y]].
```

Starting point: (-0.8, -0.6)



5. The actual top of the mountain is located at the point $(0.55652, -0.587053)$. What is the value of ∇f there? Can you explain why?