Ground Rules

- 2 types of problems
  - Drill problems
  - Problem solving problems

- Drill Problems: (6)
  - Each team sends a representative to the board
  - First team whose rep finishes first (correctly) gets **1 point**
- Problem Solving Problems (3)
  - Each team works together to solve the given problem
  - When solved correctly, the team who finishes first will brief the problem to the class, and receive **2 points**.
  - At the end, the class will vote and **one extra point** will be awarded to the team with the best briefing.

Find and sketch the domain of the function

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Answer: $D = \{(x, y) | 9 - x^2 - y^2 > 0\}$

If $f(x, y) = e^{-2xy^2}$, compute ∇f.

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Answer: $\nabla f = \langle -2y^2 e^{-2xy^2}, -4xye^{-2xy^2} \rangle$

If $f(x, y) = \dfrac{x^2 + 3xy}{x^2 + y^2}$, find $\lim\limits_{(x,y)\to(1,2)} f(x, y)$

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Answer: $\lim\limits_{(x,y)\to(1,2)} f(x,y) = \dfrac{7}{5}$

If $g(s,t) = \sqrt{s^2 + st}$, compute $\dfrac{\partial g}{\partial s}$

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Answer: $\dfrac{\partial g}{\partial s} = \dfrac{1}{2}(s^2 + st)^{-1/2} \cdot (2s + t)$

What is the rate of change of the function $f(x, y) = \sqrt{x}y + \ln(x + y)$ at the point $(1, 3)$ in the southward direction?

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Answer: This is $-f_y(1, 3) = -\left(1 + \frac{1}{4}\right) = -\frac{5}{4}$

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Answer:

$f_{xx} = 6xy^5 + 24x^2y$

$f_{yy} = 20x^3y^3$

$f_{xy} = f_{yx} = 15x^2y^4 + 8x^3$

Projectile Motion: We have derived that the range function is

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Projectile Motion: We have derived that the range function is

$$R(\theta, v_0) = \frac{v_0^2}{16}\cos(\theta)\sin(\theta).$$

- Describe in words what R_θ and R_{v_0} are measuring.
- Suppose you have a water balloon launcher set to have an angle of 30° and an initial velocity of 60 ft/sec, but you are not happy with how far it is shooting. In what 'direction' should you change θ and v_0 to ensure the maximum increase in the range of your water balloon? What will the rate of increase be?

Answers:

- R_θ measures the rate of change of the range of the projectile when v_0 is fixed and θ is allowed to vary. R_{v_0} measures the rate of change of the range of the projectile when θ is fixed and v_0 is allowed to vary.
- The direction of max increase in R is given by

$$\nabla R(30^\circ, 60) = \langle 112.5, 3.248 \rangle,$$

and the actual rate of increase is

$$|\nabla R(30^\circ, 60)| = 112.5.$$

Suppose you and your friend are each climbing a hill whose shape is given by the equation

$$z = 1000 - 0.005x^2 - 0.01y^2,$$

where x, y, and z are in meters. You and your friend are standing at two different points: $(60, y, 966)$. The positive x axis is east and the positive y axis is north.

- What are the coordinates of you and your friend?
- If each of you walk northwest, will you each start to ascend or descend? At what rate?
- For each of you, in what direction is the slope the largest? What is the rate of ascent in this direction?
- At what angle above the horizontal does the path in this direction begin?

Answers:

- Set $z = 966$ and $x = 60$ and solve to get the y coordinates. Say $(60, 40, 966)$ is your location and $(60, -40, 966)$ is your friend's location.
- (Let $f(x, y) = z$.) We find the directional derivative in the northwest direction. The unit vector in the NW direction is $\mathbf{u} = \langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$. We have

$$\nabla f = \langle -0.01x, -0.02y \rangle.$$

So for you,

$$D_{\mathrm{u}} f(60, 40) = \nabla f(60, 40) \cdot \mathbf{u} = \langle -0.6, -0.8 \rangle \cdot \mathbf{u} \approx -0.14$$

and for your friend

$$D_{\mathrm{u}} f(60, -40) = \nabla f(60, -40) \cdot \mathbf{u} = \langle -0.6, 0.8 \rangle \cdot \mathbf{u} \approx 0.99.$$

So you will descend at a rate of 0.14 meters and your friend will ascend at a rate of 0.99 meters.

- For you, the direction of largest slope is

$$\nabla f(60, 40) = \langle -0.6, -0.8 \rangle,$$

and the rate of ascent in this direction is

$$|\nabla f(60, 40)| = 1.$$

For your friend, the direction of largest slope is

$$\nabla f(60, -40) = \langle -0.6, 0.8 \rangle,$$

and the rate of ascent in this direction is

$$|\nabla f(60, -40)| = 1.$$

- Since we get the rate of ascent to be 1 in each case above, each path begins at a 45° angle above the horizontal.