

MA104 - Differential Calculus

Lesson 52: Solving Multivariate Optimization Problems I

Multivariate Optimization Steps to Success:

Step 1 Name all variables and draw a diagram (if possible).

Step 2 Determine the objective function, say Q .

Step 3 (a) Write down constraints and, if necessary, reduce Q to a function of 2 variables.

(b) Write the domain of Q and look at a plot of Q .

Step 4 Find the critical points of Q .

– *If the domain is closed and bounded*, consider all CPs and the boundary to locate the absolute max/min of Q .

– *If the domain isn't closed/bounded*, use the 2nd derivatives test to locate the max/min of Q .

Step 5 Conclude: Answer all questions that were asked in the problem, including units where necessary.

Second Derivatives Test Suppose the second partial derivatives of f are continuous on a disk with center (a, b) , and suppose that (a, b) is a critical point of f . Let

$$D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2.$$

(a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

(b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

(c) If $D < 0$, then $f(a, b)$ is a saddle point.
