MA104 - Differential Calculus
Lesson 57: Lagrange Multipliers I

Lagrange Multipliers: Find max/min values of $f(x, y, z)$ subject to $g(x, y, z) = k$.

- Find all x, y, z, and λ that solve

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

  and

$$g(x, y, z) = k$$

- Evaluate f at all (x, y, z) that you find. The largest is the max value of f; the smallest is the min value of f.

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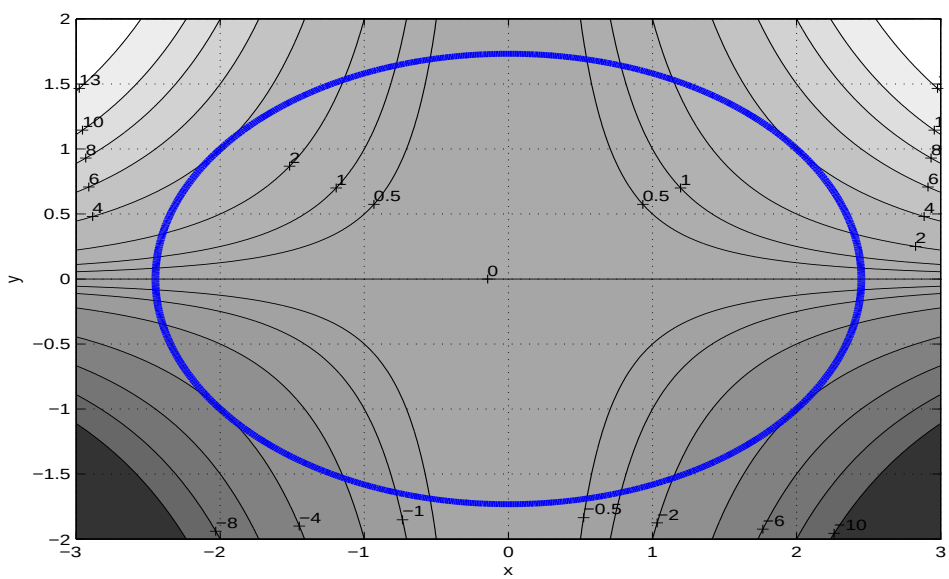
1. Let $g(x, y) = x^2 + 2y^2$ and $f(x, y) = x^2 y$. Below is a contour plot of the function $f(x, y)$. The bold curve is a graph of $g(x, y) = 6$.



   (a) By inspecting the graph, what is the maximum value of $f(x, y)$ on the curve $g(x, y) = 6$? What is the minimum value?

   (b) On the graph, plot ∇f and ∇g at the points where the extrema occur. What relationship do they have?

   (c) *By hand*, write down the system of equations that would need to be solved to use Lagrange multipliers on this problem.

2. Work problem 9 on page 940. Use this as an opportunity to build a template for Lagrange multipliers using 3 variable functions.

3. Recall the following problem from last week: Find the point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$. Re-formulate this problem in terms of Lagrange multipliers. Then solve it using this new method (and make sure your answers match!).