Applications of A Probability Programming Language (APPL)

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Outline

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Introduction

Motivation for APPL (A Probability Programming Language):

- Flexibility over statistical packages
- Research
  - mathematically intractable problems in probability
  - creating new probability distributions
- Probability education

APPL is available at no charge to non-commercial users at

`www.APPLsoftware.com`

What is APPL?

APPL (A Probability Programming Language) gives exact solutions to probability problems. Sample functions:

- `Transform` to find the transformation of a random variable;
- `Convolution` to find the sum of independent random variables;
- `Mean` to find the expected value of a random variable;
- `OrderStat` to find the distribution of an order statistic;
- `Minimum` to find the distribution of a minimum;
- `PDF`, `CDF`, `IDF`, `HF` to find the PDF, CDF, IDF, HF;
- `ExponentialRV`, and other popular distributions.

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Data structures

Continuous random variables

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 2 - x & 1 \le x < 2 \end{cases}$$

```
> X := [[x -> x, x -> 2 - x], [0, 1, 2],
        ["Continuous", "PDF"]];
```

Discrete random variables

$$f(x) = \begin{cases} 0.7 & x = 1 \\ 0.2 & x = 2 \\ 0.1 & x = 3 \end{cases}$$

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> X := [[0.7, 0.2, 0.1], [1, 2, 3], ["Discrete", "PDF"]];
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Built-in distributions and moments

```
> X := TriangularRV(0, 1, 2);
> Y := BinomialRV(10, 4 / 5);
> Z := NormalRV(mu, sigma);
> Mean(Y);
> Variance(Y);
> Skewness(Y);
> Kurtosis(Y);
```

$$8, \frac{8}{5}, -\frac{3\sqrt{10}}{20}, \frac{121}{40}$$

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Example 1: Sums of IID Random Variables

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Let $X_1, X_2, \ldots, X_{10}$ be IID U(0,1) random variables. Find

$$\Pr\left(4 < \sum_{i=1}^{10} X_i < 6\right)$$

Typical approaches

- Central limit theorem
  - population distribution not normal
  - small sample size
  - only one digit of accuracy here
- Simulation
  - requires custom programming
  - solution is given as a point and interval estimator
  - each additional digit of accuracy requires $100\times$ replications

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APPL code:

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> n := 10;
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$$\frac{655177}{907200}$$

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Let X and Y be independent random variables:

$$X \sim \mathit{Triangular}(1, 2, 4)$$

$$Y \sim \mathit{Triangular}(1, 2, 3)$$

Find the distribution of $V = XY$.

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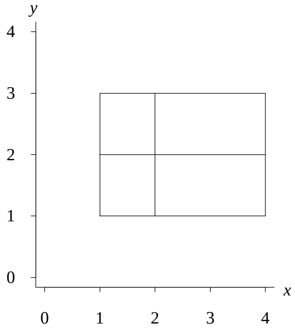

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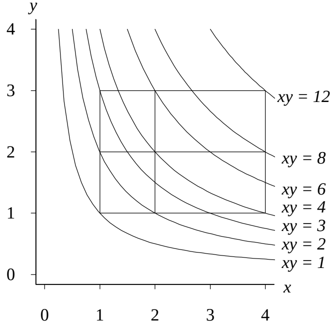

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The APPL code is

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> V := Product(X, Y);
```

$$f_V(v) = \begin{cases} -\frac{4}{3}v + \frac{2}{3}\ln v + \frac{2v}{3}\ln v + \frac{4}{3} & 1 < v \le 2 \\ -8 + \frac{14}{3}\ln 2 + \frac{7v}{3}\ln 2 + \frac{10}{3}v - 4\ln v - \frac{5v}{3}\ln v & 2 < v \le 3 \\ -4 + \frac{14}{3}\ln 2 + \frac{7v}{3}\ln 2 + 2v - 2\ln v - \\ \quad v\ln v - 2\ln 3 - \frac{2v}{3}\ln 3 & 3 < v \le 4 \\ \frac{44}{3} - 14\ln 2 - \frac{7v}{3}\ln 2 - \frac{8}{3}v - 2\ln 3 + \\ \quad \frac{22}{3}\ln v - \frac{2v}{3}\ln 3 + \frac{4v}{3}\ln v & 4 < v \le 6 \\ \frac{8}{3} - 8\ln 2 - \frac{4v}{3}\ln 2 - \frac{2}{3}v + \frac{4}{3}\ln v + \\ \quad \frac{v}{3}\ln v + 4\ln 3 + \frac{v}{3}\ln 3 & 6 < v \le 8 \\ -8 + 8\ln 2 + \frac{2v}{3}\ln 2 + \frac{2}{3}v + 4\ln 3 - \\ \quad 4\ln v + \frac{v}{3}\ln 3 - \frac{v}{3}\ln v & 8 < v < 12 \end{cases}$$

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Example 3: Order Statistics

A bag contains 15 billiard balls, numbered 1 to 15. If 7 balls are drawn from the bag at random, find the probability that the median number drawn is 5 when (a) sampling is performed without replacement; (b) sampling is performed with replacement.

(a) Sampling without replacement

```
> X := UniformDiscreteRV(1, 15);
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> PDF(Y, 5);
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$$\frac{32}{429}$$

Example 3: Order Statistics

A bag contains 15 billiard balls, numbered 1 to 15. If 7 balls are drawn from the bag at random, find the probability that the median number drawn is 5 when (a) sampling is performed without replacement; (b) sampling is performed with replacement.

(a) Sampling without replacement

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Rose–Hulman
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$$\frac{2949971}{34171875}$$

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APPL Tree

- APPL Procedures
  - Functional forms
    - `PDF`
    - `CDF`
    - `SF`
    - `HF`
    - `CHF`
    - `IDF`
  - Procedures on One Random Variable
    - `Transform`
    - `OrderStat`
    - `Truncate`
    - `ConvolutionIID`
    - `ProductIID`
    - `ExpectedValue`
      - `Mean`
      - `Variance`
      - `Skewness`
      - `Kurtosis`
      - `MGF`
      - `CF`
  - Procedures on Two Random Variables
    - `Convolution`
    - `Product`
    - `Minimum`
    - `Maximum`
    - `Mixture`
  - Statistical Procedures
    - `MLE`
    - `MOM`
    - `MLENHPP`
    - `KSTest`
    - `QQPlot`
    - `PPPlot`
  - Utilities
    - `VerifyPDF`
    - `PlotDist`
      - `PlotEmpCDF`
      - `PlotEmpSF`
      - `PlotEmpCIF`
      - `PlotEmpVsFittedCDF`
      - `PlotEmpVsFittedSF`
      - `PlotEmpVsFittedCIF`
    - `Menu`
    - `Display`
    - `CleanUp`