3D-Analysis – Documentation

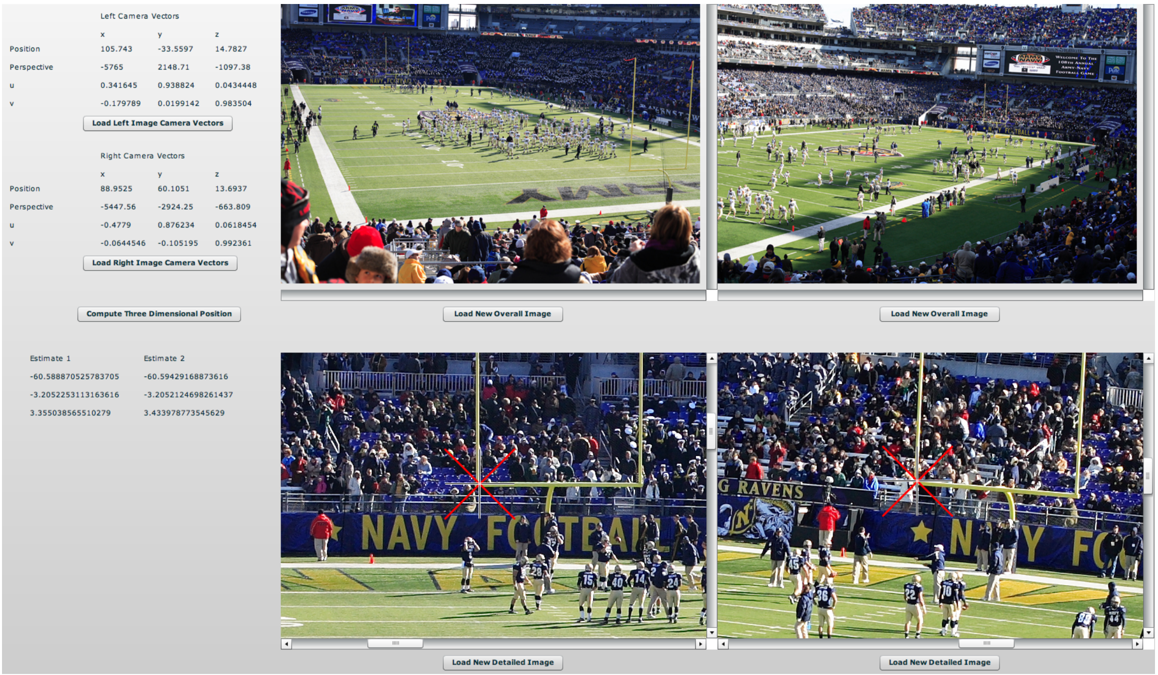

Figure 1: Estimating the Real World 3D Coordinates of a Feature from Two Images

This program enables you to estimate the real world three dimensional coordinates of a feature from two images. The screenshot in Figure 1 illustrates this using two images taken at the December 2007 Army-Navy game. The two images were used to estimate the location of a particular point on one end of the crossbar on the goalpost at the far end of the field. We obtained an estimate of $(-60.6, -3.2, 3.4)$ measured in yards with the origin at the center of the field. Figure 2 on page 2 shows the real world coordinate axes. Figure 3 on page 4 shows the specifications for a football field. According to those specifications the coordinates of the particular point should be $(-60, -3.06, 3.33)$. Our estimates from the same two photographs for the location of the other end of the cross bar are $(-60.7, 2.95, 3.48)$ and according to the specifications this point should be $(-60, 3.06, 3.33)$. As you can see these estimates are quite good – "in the ballpark" – especially since the photographs were taken from the stands over 150 yards from the goalposts.

The screenshot in Figure 1 shows two images from each of the two photographs. The top image is an overall image, showing the entire photograph. The bottom image is a close up showing all the detail from a small section of the full photograph. The user first clicks close to the feature of interest in the top image. The bottom image then shows the area

Figure 2: Real World Coordinate Axes at the Army-Navy Game

around the feature. The user then is able to click more precisely on the feature in the bottom image. The user repeats this for the second photograph. Finally the user clicks the **Compute Three Dimensional Position Button**. Two estimates for the three dimensional coordinates of the feature are then shown. The average of the two is the best estimate.

You are now ready to use 3D-Image-Analysis. Download the file **3D-Analysis.zip** and unzip it. You should see a directory with several files including the following.

- **3D-Analysis.html** – This is the program. Note that it requires FlashPlayer. This file runs within your browser. On some WindowsOS computers Internet Explorer does not work. We recommend FireFox.

- **AN76.xml** – This file contains the calibration data for one of the two photographs. This file is discussed in more detail in the documents **3D Image Calibration –** *Mathematica* **Documentation** and **3D Image Calibration – Excel Documentation**.

- **AN77.xml** – This file contains the calibration data for the second of the two photographs.
- **AN76-detailed** – This is the original full-sized digital image for the first photograph. You can use any digital image in JPG format.
- **AN77-detailed** – This is the original full-sized digital image for the second photograph.
- **AN76-overview** – This is the smaller digital image for the first photograph. You can use a standard digital image processing program to reduce your original full-sized image to an image that is no more than 600 pixels wide and no more than 400 pixels high.
- **AN77-overview** – This is the smaller digital image for the second photograph.

When you work with your own photographs all seven of these files must be in the same directory. You may use your own names for the image and calibration files.

- Double-click the file **3D-Analysis.html**. Note that it requires FlashPlayer. This file runs within your browser. On some WindowsOS computers Internet Explorer does not work. We recommend FireFox.
- Click the **Load New Overall Image** button under the upper left of the four panels for the four images. In the browse window that opens navigate to and select the file **AN76-overview.jpg**.
- Click the **Load New Detailed Image** button under the lower left of the four panels for the four images. In the browse window that opens navigate to and select the file **AN76-detailed.jpg**.
- Click the **Load New Overall Image** button under the upper right of the four panels for the four images. In the browse window that opens navigate to and select the file **AN77-overview.jpg**.
- Click the **Load New Detailed Image** button under the lower right of the four panels for the four images. In the browse window that opens navigate to and select the file**AN77-detailed.jpg**.
- Click the **Load Left Image Camera Vectors** button in the upper left of the screen. In the browse window that opens navigate to and select the file **AN-76.xml**.
- Click the **Load Right Image Camera Vectors** button. In the browse window that opens navigate to and select the file **AN-77.xml**.

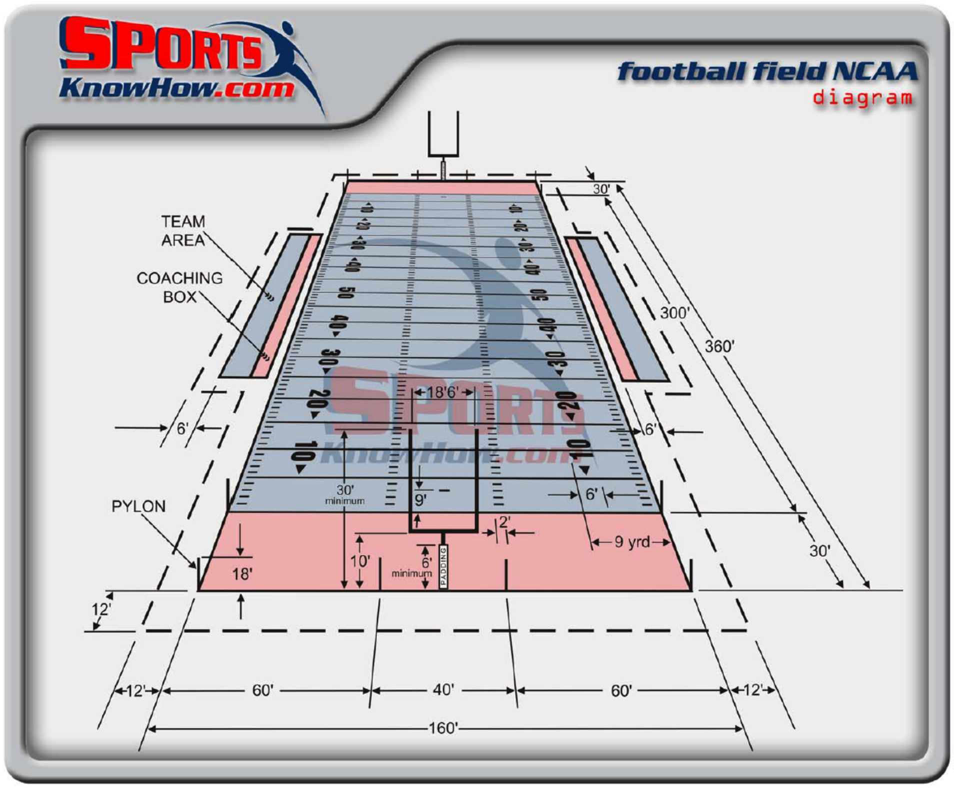

Figure 3: Football Field Diagram – from SportsKnowHow.com

- Now you can estimate the real world 3D-coordinates of any recognizable feature that is shown in both photographs as described above.

The Mathematics Behind this Analysis

The key to this analysis is the calibration data, or the camera vectors, that describe how the photographs were taken. There are four vectors.

- A vector $\vec{q}$ that describes in real world coordinates the location of the optical center of the camera lens.
- A vector $\vec{p}$ that describes the direction in which the camera was pointing and a magnification factor that is determined by the focal length of the lens, the dimensions of the camera's sensor, and the dimensions (in pixels) of the photograph.
- A unit vector $\vec{u}$ that points in the positive x-direction of the image on the camera sensor.

- A unit vector $\vec{v}$ that points in the positive y-direction of the image on the camera sensor.

We use these vectors to convert back-and-forth between the real world coordinates $\vec{a} = (x, y, z)$ of a point (feature) and its coordinates (s, t) on the photograph. The translation from real world coordinates to photograph coordinates is given by

$$\begin{aligned} \vec{h} &= \text{proj}_{\vec{p}}\,(\vec{a} - \vec{q}) = \left(\frac{1}{||\vec{p}||^2}\right)((\vec{a} - \vec{q}) \cdot \vec{p})\vec{p} \\ s &= \left(\frac{||\vec{p}||}{||\vec{h}||}\right)((\vec{a} - \vec{q} - \vec{h}) \cdot \vec{u}) \\ t &= \left(\frac{||\vec{p}||}{||\vec{h}||}\right)((\vec{a} - \vec{q} - \vec{h}) \cdot \vec{v}) \end{aligned}$$

If we know the real world coordinates of a point (feature) in three space then we know its coordinates on the photograph but if we know its coordinates on the photograph then all we know is that its real world coordinates lie on the line given by

$$\vec{f}(r) = \vec{q} + r(\vec{p} + s\vec{u} + t\vec{v}).$$

Now suppose we have the same point (feature) in two photographs. We use the notation $\vec{q}_1, \vec{p}_1, \vec{u}_1$, and $\vec{v}_1$ for the camera vectors for the first photograph and $\vec{q}_2, \vec{p}_2, \vec{u}_2$, and $\vec{v}_2$ for the camera vectors for the second photograph. We use the notation (s_1, t_1) for the feature's coordinates on the first photograph and (s_2, t_2) for its coordinates on the second photograph. Then we know, in theory, that the point lies on the intersection of the two lines

$$\vec{f}_1(r_1) = \vec{q}_1 + r_1(\vec{p}_1 + s_1\vec{u}_1 + t_1\vec{v}_1) \quad \text{and} \quad \vec{f}_2(r_2) = \vec{q}_2 + r_2(\vec{p}_2 + s_2\vec{u}_2 + t_2\vec{v}_2).$$

In practice, however, there will be some measurement and round-off error and as a result these two lines usually will not intersect. Thus, we need to minimize the function

$$\mathcal{E}(r_1, r_2) = ||\vec{f}_1(r_1) - \vec{f}_2(r_2)||.$$

This is exactly how the program **3D-Analysis** arrives at the two estimates. Given $\bar{r}_1$ and $\bar{r}_2$ that minimize $\mathcal{E}(r_1, r_2)$, the two estimates are

$$\vec{f}_1(\bar{r}_1) \quad \text{and} \quad \vec{f}_2(\bar{r}_2)$$

and the best estimate is the average of these two estimates.

The mathematics and geometry behind this flurry of formulas is discussed in the paper **The Mathematics of Working with Digital 3D Imagery**.