Value of Information in a Competitive Game

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Abstract: *We develop basic principles, metrics, and concepts related to modeling the acquisition and use of information in a simple matrix game. We apply this model to a battlefield game using the Mixed Law of the Lanchester equations to measure effective game values. By analyzing several scenarios, we develop a useful measure for the value of information that a military commander can use. The results show that there is value in the information gained by partitioning the matrix game based on information sets and more finely partitioned information sets provide more valuable information. The specific game is programmed in Microsoft Excel so that the model can be modified to tailor the strategies and payoff structure based on the needs of the Army Research Office.*
(Game Theory; Two-Person, Zero-Sum Games; Linear Programming; Lanchester Equations)

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1 Background

Military commanders have many decisions to make during an operation. One factor of critical importance in making decisions is the use of intelligence, or knowledge of the enemy forces and environmental factors. This intelligence is the information that is significant to the planning and conducting of the operation. How do we measure the significance of information in this context? What is the value of information in a competitive game, like combat (or sports, board games, etc.)?

In order to gather this information about the enemy, a commander must incur some cost. In a military context this cost is normally strength in the form of manpower or equipment. Just as in economics, in a military operation, there are cost versus reward trade-off curves. A commander may be willing to devote significant resources to intelligence gathering to learn everything about his enemy's strategies. However, he will have fewer assets to send into combat, as these resources are being used to gather information. Conversely, if a commander does not commit assets to intelligence gathering, he risks a lower reward or chance of victory. He retains all of his strength, but he may allocate it improperly since he does not have any information about his enemy.

This analogy to economics leads one to believe that an important theory in economics, Game Theory, may be applicable to a military operation. Combat pits two armies against each other in a competitive "game." One could argue that what one army loses, the other army gains, which results in a two-person, zero-sum game.

Based on some preliminary work presented previously, the Army Research Office (ARO) believes that an application for the value of information exists in the Army. At this point, researchers are too involved in current projects to explore new approaches in the handling of information. For that reason ARO wants to explore a model that they can distribute to the research community for application in future military projects involving information.

In this paper, we developed basic principles, metrics, and concepts through the use of matrix games in a game theoretic study. We believe that there are important applications for our measure of the value of information in models presented here. In future work, we hope to continue the development of the problem contained in this paper or tailor the strategies and payoff structure based on the needs of ARO or other Army agencies.

2 Two-Person, Zero-Sum Matrix Games

In a perfect information, two-person, zero-sum matrix game there is a row player and a column player; both players have perfect information about the structure and payoffs of the game. The strategies of these two players make up make a matrix game $(\mathcal{A}, \mathcal{R}, \mathcal{C})$, where $\mathcal{A}$ is the game matrix, $\mathcal{R}$ is the set of m row strategies, and $\mathcal{C}$ is the set of n column strategies; the entries a_{ij} of the matrix represent the payoff for each player when the row player plays strategy $i \in \mathcal{R}$ and the column player plays strategy $j \in \mathcal{C}$. A zero-sum game is a special type of game in which only one payoff value is necessary. In this special type of game, what one player loses, the other gains and vice versa. Thus, the gains and losses sum to zero. Normally, payoffs in this type of a matrix game represent what the column player ("she") pays the row player ("he"). Therefore, if a_{ij} is positive, the row player gains that amount from the column player when strategies i and j are played. Conversely, if a_{ij} is negative, the row player loses that amount to the column player when strategies i and j are played.

This zero-sum characteristic of the game makes modeling and solving the game easy. We can use the game matrix $\mathcal{A}$ to define a linear program. By solving the linear program, we are looking for the set of row player maximin mixed strategies $p = (p_1, ..., p_m)$ that maximizes the minimum of the expected values of the n columns and the set of column player minimax mixed strategies $q = (q_1, ..., q_n)$ that minimizes the maximum of the expected values of the m rows. The expected value of column j, v_j^c, is the column player's expected payoff when using strategy j over the row player's maximin mixed strategy p: $v_j^c = \sum_{i=1}^{m} a_{ij} * p_i$. The expected value of row i, v_i^r, is the row player's expected payoff when using strategy i over the column player's minimax mixed strategy q: $v_i^r = \sum_{j=1}^{n} a_{ij} * q_j$.

In a mixed strategy, a player will select more than one strategy and the probability distribution p or q represents the probability in the long run that he/she will select that strategy. For example, when $q_j = x \quad \text{for } j \in \{1, ..., n\}$, the column player will select strategy j $(x * 100)$percent of the time in the long run. A pure strategy is a special type of mixed strategy where a player selects that strategy 100% of the time or every time he/she plays the game. In the case of a pure strategy, one element of p or q will be equal to 1 while all other elements of that vector will equal 0. For example, when $p_i = 1 \quad \text{for } i \in \{1, ..., m\}$, the row player will select strategy i every time he plays the game.

The optimal mixed strategy guarantees the row player an expected gain of at least: $\underline{v} = \min\{v_1^c(p), v_2^c(p), ..., v_n^c(p)\}$. This solution also guarantees the column player an expected loss of at most: $\overline{v} = \max\{v_1^r(q), v_2^r(q), ..., v_m^r(q)\}$. The linear program for the row player is:

$$\text{maximize}\, w = \min\{v_1^c(p), v_2^c(p), ..., v_n^c(p)\}$$

$$\text{subject to}: p = (p_1, ..., p_m) \text{ being a probability distribution.}$$

The linear program for the column player is:

$$\text{minimize}\, z = \max\{v_1^r(q), v_2^r(q), ..., v_n^r(q)\}$$

$$\text{subject to}: q = (q_1, ..., q_n) \text{ being a probability distribution.}$$

It easily can be shown that these are dual linear programs. Therefore, by the duality theorem, the maximin and the minimax game values will be the same (i.e., $\underline{v} = \overline{v} =$ value of the game).

3 Finding the Value of Information – one-sided multi-probe case

Information in this context refers to the information that one player learns about a particular component of the strategy that his/her opponent chooses. One player will pay some type of cost to learn the exact strategy or some component of the opponent's strategy, allowing the first player to make a strategy choice resulting in a higher payoff. The cost of this information will be in the form of a reduced actual payoff.

Information in this game is modeled as follows: The row player has the option of employing several informational probes $\mathcal{P}_0, ..., \mathcal{P}_u$. Each $\mathcal{P}_j$ is defined by a partition $\mathcal{J}_j = (\mathcal{J}_1^j, ..., \mathcal{J}_{c_j}^j)$ of the column space, together with the row space $\mathcal{R}$. The set of probes could include the null probe $\mathcal{P}_0$, with the partition $\mathcal{J}_0 = (\mathcal{C})$ and $\mathcal{R}_0 = (\mathcal{R})$, which represents the option of not employing a probe at all. Although not considered in this case, similar probes can be defined for the column player consisting of probes $\mathcal{Q}_0, ..., \mathcal{Q}_v$, with the associated column space $\mathcal{C}$ and row space partitions $\mathcal{I}_i = (\mathcal{I}_1^i, ..., \mathcal{I}_{r_i}^i)$. Each player has the option of using exactly one of the given probes, the use of which has some cost to the player.

For clarity and we introduce the probes for our model now. Our model uses the null probe $\mathcal{P}_0$, a tactic probe, $\mathcal{P}_t$, and a strength probe, $\mathcal{P}_s$. If the row player selects to employ one of these informational probes $\mathcal{P}_j$, it costs in that a specified game value penalty is attached to using a probe. In return, however, he gains the information as to which of the partitions in $\mathcal{J}_j$ the actual strategy the column player has chosen lies in. For example, suppose the use of probe $\mathcal{P}_t$ imposes a penalty on the row player (which must then necessarily be "paid" to the column player) of $x_{ij}^t > 0$ whenever the row player subsequently plays strategy i and the column player plays strategy j. Let $\mathcal{A}^t$ be the resulting matrix of penalties. The probe-partitioned game matrix then looks like

$$\begin{bmatrix} \mathcal{A} \\ \mathcal{A}^t \\ \mathcal{A}^s \end{bmatrix}$$

where the top set I_0 of rows corresponds to the null probe, with $\mathcal{R}_0 = I_0$, and the row space $\mathcal{R}$ associated with using probe t is the set of rows corresponding to the matrix of penalties $\mathcal{A}^t$. Both players know the probes are available to the other player, and choose their strategies accordingly.

We consider the case where the row player is the only player who has probes available. When multiple probes are available, then he must first decide which probe to employ. If probe $\mathcal{P}_j$ is used, then after deciding which partition element $\mathcal{J}_k^j$ the column player's strategy is in, he picks the appropriate strategy among the strategies available to him when using that probe. The row player's maximin mixed strategy can be computed as follows: Let ρ_j be the probability of him picking probe $\mathcal{P}_j$. For each $k = 1, ..., c_j$, let $p^{j,k}$ be the maximin mixed strategy for the game associated with the submatrix $\mathcal{A}^{\mathcal{R},\mathcal{J}_k^j}$ of $\mathcal{A}^j$ consisting of rows corresponding to $\mathcal{R}$ and columns corresponding to $\mathcal{J}_k^j$. Now for each row probe index j and each column strategy l, define

$$\mathcal{K}_j(l) = \{k : l \in \mathcal{J}_j^k\}.$$

The probabilities ρ_j are then computed by finding the maximin strategy corresponding to the following s x n game matrix that we call the composite game:

$$\mathcal{A}^p = \begin{bmatrix} p^{1,\mathcal{K}_1(1)}\mathcal{A}^{\mathcal{R},1} & \dots & p^{1,\mathcal{K}_1(n)}\mathcal{A}^{\mathcal{R},n} \\ \vdots & & \vdots \\ p^{s,\mathcal{K}_s(1)}\mathcal{A}^{\mathcal{R},1} & \dots & p^{s,\mathcal{K}_s(n)}\mathcal{A}^{\mathcal{R},n} \end{bmatrix}.$$

The row player then chooses to use probe $\mathcal{P}_j$ with probability ρ_j, and if the probe is used and partition $\mathcal{J}_k^j$ is observed, then the mixed strategy $p^{j,k}$ is used.

The column player computes her minimax column strategy by simply computing the minimax strategy $q = (q_1, ..., q_n)$ on the same matrix $\mathcal{A}^p$ used by the row player. The vector of probabilities then represents her minimax strategy for the original multi-probe game.

4 Lanchester Equation Game Model

4.1 Model Description

This project is based on a battlefield that is divided into two distinct and separate geographical sectors similar to a battlefield divided by a mountain or large body of water. The players allocate their troops to one sector or the other. Troops allocated to one sector cannot engage troops in the other sector. The key assumptions for this model applying to all games and sub-games are:

- each player must allocate at least 10% of their overall strength to each sector
- no reserves; each player must allocate 100% of their overall troop strength
- each player can only allocate troops in increments of 10%

The Lanchester equations are widely accepted as valid combat models in theoretic applications for modeling attrition. In this game with the Lanchester equations, the Blue player will be the row player and his strategies will correspond to $\mathcal{R}$. The Red player will be the column player and her strategies will correspond to $\mathcal{C}$. Per engagement, each player or commander can choose a tactic (defend or attack) and a strength allocation for the engagement. Tactic is represented in the Excel file model as $T \in \{1, 2\}$ respectively. Each player begins with the total overall strength B for the Blue player and R for the Red player. Strength allocation is represented in the Excel file model as $S \in \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$. Strength allocation, S, is defined as the percentage of overall strength allocated to sector 1. From the assumptions, whatever available troop strength that is not allocated to sector 1 as indicated by S, must be allocated to sector 2. For example, if B_1 is defined as Blue's troop strength in sector 1, then $B_1 = B * S$. Likewise, if R_2 is defined as Red's troop strength in sector 2, then $R_2 = R * (100 - S)$.

Strategies for the Blue player are defined as $i = (T_b, S_b)$ and strategies for the Red player are defined as $j = (T_r, S_r)$. The two competitors' choice of tactic and strength allocation determine the payoff, a_{ij}, and thus the winner of each engagement through computations using the Lanchester equations. Each player has the option of 2 tactics and 9 strength allocations for a total of 18 strategies each.

4.2 The Lanchester Equations

Lanchester equations are differential equations describing the attrition of Blue and Red strengths B and R as a function of time, with the function depending only on the current values of B and R. The general form of the equations is:

$$\frac{dB}{dt} = -K_r B^e R^g \qquad \frac{dR}{dt} = -K_b R^f B^h.$$

K_i is the attrition coefficient or the rate at which force i kills the other force (Davis, 1995). There are two special cases of the Lanchester equations discussed in most relevant literature. The Linear Law corresponds to $e = f = g = h = 1$ and applies to instances with unknown precise locations, area fire, or ancient warfare (Davis, 1995; Darilek, Perry, Bracken, Gordon, & Nichiporuk, 2001). The Square Law corresponds to $g = h = 1$ and $e = f = 0$ and applies to instances with known precise locations, aimed fire, or modern warfare.

$$\text{LinearLaw}: \quad \frac{dB}{dt} = -K_r BR \qquad \frac{dR}{dt} = -K_b RB$$

$$\text{SquareLaw}: \quad \frac{dB}{dt} = -K_r R \qquad \frac{dR}{dt} = -K_b B$$

In modeling this game we used a special Mixed Law as developed by the RAND Corporation (Darilek et al., 2001). The Mixed Law is an adaptation of the Square Law to account for a level of knowledge of enemy troop location.

$$\text{MixedLaw}: \quad \frac{dB}{dt} = -c_r K_r R \qquad \frac{dR}{dt} = -c_b K_b B$$

The authors defined c_r and c_b as a function representing external information available to Red and Blue respectively. In a similar fashion, we use these variables but define them to be the increase in efficiency due to information gained by using scouts or external sources. For instance, if Blue using a scout increases Blue's ability to kill Red by 40%, then $c_b = 1.4$. If there is no information gain, then $c_b = 1$. Typically, c values can range between 1 and 2 for this model.

Specific to this game model, K can also depend on the tactic that the player chooses and which sector the engagement is taking place in. For instance, K_{ra1} is the rate at which the Red force kills Blue when Red is attacking in sector 1, and K_{bd2} is the rate at which the Blue force kills Red when Blue is defending in sector 2. K_{ra1} may be higher than K_{ra2} if say sector 1 has more avenues of approach and is better suited for Red to attack. Typically, K_{bd2} would be higher than K_{ba2} because in a defense, Blue would have the advantages of surprise, cover and concealment, and fewer moving pieces. However, K_{ba2} may be higher than K_{bd2} if perhaps sector 2 is barren, full of avenues of approach, and there is little time to prepare a defense. It is left to the user to specify these K values based on the situation he is modeling. $K \in (0, 1)$ and typically $K \in [0.08, 0.3]$ for an attacking force and $K \in [0.2, 0.5]$ for a defending force depending on terrain, training, morale, weaponry, and capabilities.

4.3 The Model as a Tool

Regardless of the law, K values, and c values, we define the Blue loss rate as $BLR = (dB/dt)/B$ and Red loss rate as $RLR = (dR/dt)/R$. From the loss rates, we define the difference of loss rates as $DOLR = BLR - RLR = a_{ij}$. So, $DOLR$s are the payoffs in the game matrix. Because the loss rates are negative numbers, each player wants his/her own loss rate to be as close to 0 as possible. It follows from this logic that when $DOLR$ is less than 0, Red is winning the engagement, and when $DOLR$ is greater than 0, Blue is winning the engagement. Therefore, Blue is selecting a strategy that will maximize the minimum expected $DOLR$ and Red is selecting a strategy that will minimize the maximum expected $DOLR$. Table 1 is a summary of all the Lanchester equation definitions used in this model. Some of the variables will be further explained when discussing information.

As a result of the Mixed Law of the Lanchester equations, the definition of the model, and the definitions in Table 1, the payoffs for the main game, a_{ij}, are calculated as follows:

$$
DOLR \begin{array}{cl} = & BLR - RLR = \frac{\frac{dB}{dt}}{B} - \frac{\frac{dR}{dt}}{R} \\ = & \frac{-K_r * R}{B} - \frac{-K_b * B}{R} \\ = & \frac{K_b * B}{R} - \frac{K_r * R}{B} \\ = & \left(\frac{K_{bT_b 1} * (B * S_b * 0.01)}{R * S_r * 0.01} + \frac{K_{bT_b 2} * [B * (100 - S_b) * 0.01]}{R * (100 - S_r) * 0.01}\right) - \left(\frac{K_{rT_r 1} * (R * S_r * 0.01)}{B * S_b * 0.01} + \frac{K_{rT_r 2} * [R * (100 - S_r) * 0.01]}{B * (100 - S_b) * 0.01}\right). \end{array}
$$

Table 1: Lanchester Equation Definitions

Symbol	**Definition**
B	Blue strength; **Blue is row player**
R	Red strength; **Red is column player**
K_{bd1}	Attrition coefficient; rate at which Blue kills Red when Blue is defending in sector 1
K_{bd2}	Attrition coefficient; rate at which Blue kills Red when Blue is defending in sector 2
K_{ba1}	Attrition coefficient; rate at which Blue kills Red when Blue is attacking in sector 1
K_{ba2}	Attrition coefficient; rate at which Blue kills Red when Blue is attacking in sector 2
K_{rd1}	Attrition coefficient; rate at which Red kills Blue when Red is defending in sector 1
K_{rd2}	Attrition coefficient; rate at which Red kills Blue when Red is defending in sector 2
K_{ra1}	Attrition coefficient; rate at which Red kills Blue when Red is attacking in sector 1
K_{ra2}	Attrition coefficient; rate at which Red kills Blue when Red is attacking in sector 2
T_b	Tactic chosen by the Blue player; defend=d=1 and attack=a=2
T_r	Tactic chosen by the Red player; defend=d=1 and attack=a=2
S_b	Percentage of strength Blue allocates to sector 1
S_r	Percentage of strength Red allocates to sector 1
X	Cost of employing tactic scouts
Y	Cost of employing strength scouts
c_{bx}	Blue's increase in efficiency due to information from the tactic scouts
c_{by}	Blue's increase in efficiency due to information from the strength scouts
BLR	Blue loss rate; $(dB/dt)/B$
RLR	Blue loss rate; $(dR/dt)/R$
$DOLR$	Difference of loss rates; $BLR - RLR$; a_{ij}

The Excel file model calculates all of the payoffs for all games and sub-games automatically with simple changes of any of the variables defined above (highlighted yellow and located towards the top in rows 4, 7, 8, 11, 12, 15, and 16 of the first worksheet labeled "Composite Game") (Figure 1). Due to these automatic calculations, there is no need to manipulate any entries in the (18 x 18) matrix $\mathcal{A}$ (Figure 2) or any cells not highlighted yellow for that matter.

Using the theory discussed in Section 2, we have used the model in Figure 2 to solve for the optimal set of mixed strategies for both the Blue player and the Red player. The optimal mixed strategy for the Blue player, p, which maximizes his minimum expected payoff, is located in column W of Figure 2; the optimal mixed strategy for the Red player, q, which minimizes her maximum expected payoff, is located in row 29 of Figure 2. The game value is located in cells W30 and X29. The model automatically applies the theory and solves many linear programs for all games and sub-games simultaneously when the user clicks the "Solve Game" button located in columns A-C and rows 28-29 on the “Composite Game” worksheet as shown in Figure 1. The “Solve” button located near columns A-B and rows 5-7 (Figure 2) of the main game and sub-game worksheets only solves the game on that respective worksheet. The “Solve” buttons are primarily diagnostic tools for the designer or someone modifying the model. Please refer to Appendix A for instructions on loading the model and Appendix B for instruction on using the model.

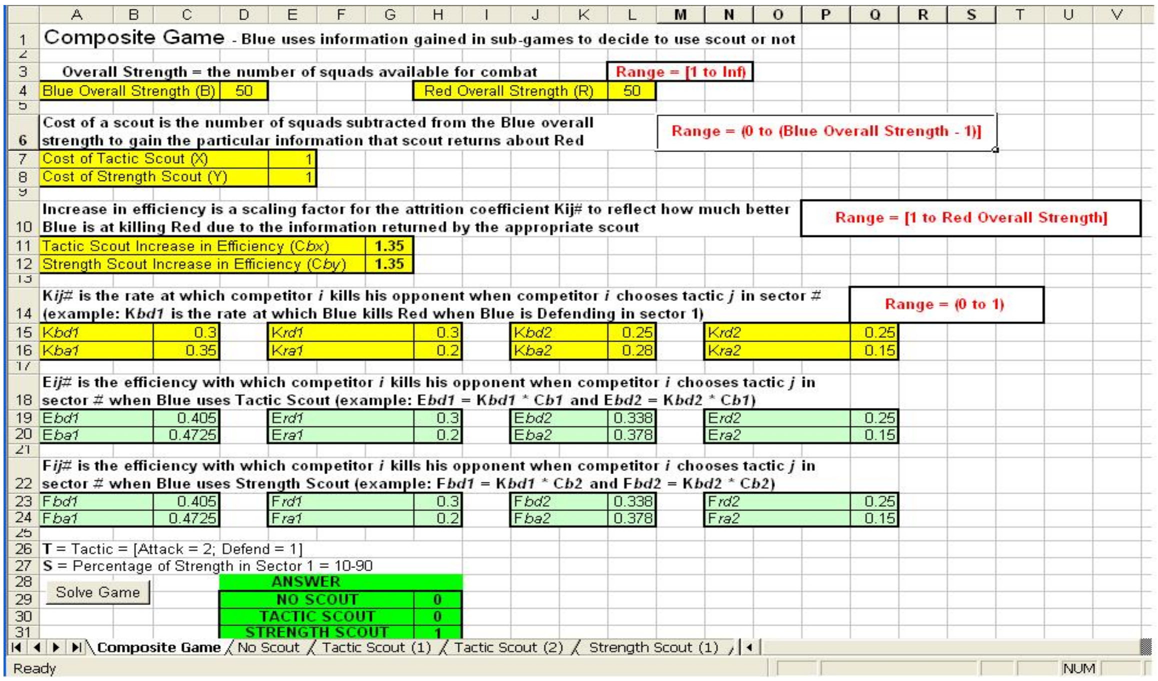
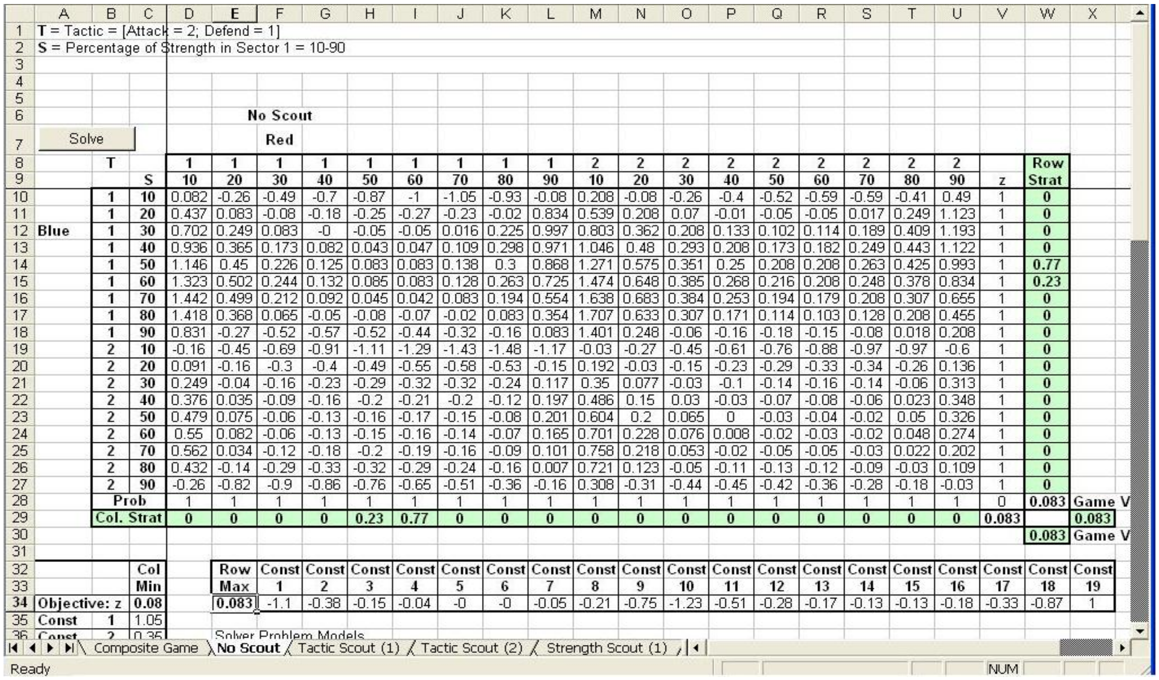


Figure 1: Excel File Model - Variables on Composite Game Worksheet

5 Information Model

Since our model involves a military example, we refer to probes as scouts. In this model, the Blue player (the row player) is the only competitor capable of gaining information. For simplification of the model and ease of demonstration, we assume that each player must apply the chosen tactic to both sectors. We also assume that troops committed to scouting cannot participate in the engagement. This assumption is necessary to account for the cost of using scouts. Also for this model, we assume that the information gathered by the scouts is 100% accurate. Relaxing this assumption is an area for future work. As a means of scaling, we consider one troop strength equal to one squad of troops.

The Blue player has three types of scouts available to him: no scouts, tactic scouts, or strength scouts. As discussed in Section 3, there must be a penalty or cost attached to using one of these scouts. The cost of information is the number of squads subtracted from B, the overall troop



T = Tactic = [Attack = 2; Defend = 1]
S = Percentage of Strength in Sector 1 = 10-90

Solve

No Scout
Red

	T		1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2		Row
		S	10	20	30	40	50	60	70	80	90	10	20	30	40	50	60	70	80	90	z	Strat
	1	10	0.082	-0.26	-0.49	-0.7	-0.87	-1	-1.05	-0.93	-0.08	0.208	-0.08	-0.26	-0.4	-0.52	-0.59	-0.59	-0.41	0.49	1	0
	1	20	0.437	0.083	-0.08	-0.18	-0.25	-0.27	-0.23	-0.02	0.834	0.539	0.208	0.07	-0.01	-0.05	-0.05	0.017	0.249	1.123	1	0
Blue	1	30	0.702	0.249	0.083	-0	-0.05	-0.05	0.016	0.225	0.997	0.803	0.362	0.208	0.133	0.102	0.114	0.189	0.409	1.193	1	0
	1	40	0.936	0.365	0.173	0.082	0.043	0.047	0.109	0.298	0.971	1.046	0.48	0.293	0.208	0.173	0.182	0.249	0.443	1.122	1	0
	1	50	1.146	0.45	0.226	0.125	0.083	0.083	0.138	0.3	0.868	1.271	0.575	0.351	0.25	0.208	0.208	0.263	0.425	0.993	1	0.77
	1	60	1.323	0.502	0.244	0.132	0.085	0.083	0.128	0.263	0.725	1.474	0.648	0.385	0.268	0.216	0.208	0.248	0.378	0.834	1	0.23
	1	70	1.442	0.499	0.212	0.092	0.045	0.042	0.083	0.194	0.554	1.638	0.683	0.384	0.253	0.194	0.179	0.208	0.307	0.655	1	0
	1	80	1.418	0.368	0.065	-0.05	-0.08	-0.07	-0.02	0.083	0.354	1.707	0.633	0.307	0.171	0.114	0.103	0.128	0.208	0.455	1	0
	1	90	0.831	-0.27	-0.52	-0.57	-0.52	-0.44	-0.32	-0.16	0.083	1.401	0.248	-0.06	-0.16	-0.18	-0.15	-0.08	0.018	0.208	1	0
	2	10	-0.16	-0.45	-0.69	-0.91	-1.11	-1.29	-1.43	-1.48	-1.17	-0.03	-0.27	-0.45	-0.61	-0.76	-0.88	-0.97	-0.97	-0.6	1	0
	2	20	0.091	-0.16	-0.3	-0.4	-0.49	-0.55	-0.58	-0.53	-0.15	0.192	-0.03	-0.15	-0.23	-0.29	-0.33	-0.34	-0.26	0.136	1	0
	2	30	0.249	-0.04	-0.16	-0.23	-0.29	-0.32	-0.32	-0.24	0.117	0.35	0.077	-0.03	-0.1	-0.14	-0.16	-0.14	-0.06	0.313	1	0
	2	40	0.376	0.035	-0.09	-0.16	-0.2	-0.21	-0.2	-0.12	0.197	0.486	0.15	0.03	-0.03	-0.07	-0.08	-0.06	0.023	0.348	1	0
	2	50	0.479	0.075	-0.06	-0.13	-0.16	-0.17	-0.15	-0.08	0.201	0.604	0.2	0.065	0	-0.03	-0.04	-0.02	0.05	0.326	1	0
	2	60	0.55	0.082	-0.06	-0.13	-0.15	-0.16	-0.14	-0.07	0.165	0.701	0.228	0.076	0.008	-0.02	-0.03	-0.02	0.048	0.274	1	0
	2	70	0.562	0.034	-0.12	-0.18	-0.2	-0.19	-0.16	-0.09	0.101	0.758	0.218	0.053	-0.02	-0.05	-0.05	-0.03	0.022	0.202	1	0
	2	80	0.432	-0.14	-0.29	-0.33	-0.32	-0.29	-0.24	-0.16	0.007	0.721	0.123	-0.05	-0.11	-0.13	-0.12	-0.09	-0.03	0.109	1	0
	2	90	-0.26	-0.82	-0.9	-0.86	-0.76	-0.65	-0.51	-0.36	-0.16	0.308	-0.31	-0.44	-0.45	-0.42	-0.36	-0.28	-0.18	-0.03	1	0
	Prob		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0.083 Game V
	Col. Strat		0	0	0	0	0.23	0.77	0	0	0	0	0	0	0	0	0	0	0	0	0.083	0.083
																						0.083 Game V

		Col	Row	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const
		Min	Max	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Objective: z		0.08	0.083	-1.1	-0.38	-0.15	-0.04	-0	-0	-0.05	-0.21	-0.75	-1.23	-0.51	-0.28	-0.17	-0.13	-0.13	-0.18	-0.33	-0.87	1
Const	1	1.05																				

Solver Problem Models

Composite Game / No Scout / Tactic Scout (1) / Tactic Scout (2) / Strength Scout (1)

Ready

Figure 2: Excel File Model - No Scout Main Game Worksheet

strength for Blue. X, the cost of using tactic scouts, is the maximum number of squads the Blue player is willing to sacrifice from his overall troop strength to determine which tactic Red is using in both sectors. Y, the cost of using strength scouts, is the maximum number of squads the Blue player is willing to sacrifice from his overall troop strength to determine how Red has allocated his troop strength among the sectors.

Choosing no scouts costs the Blue player nothing and $\mathcal{P}_0 = (\mathcal{R}, \mathcal{C})$ using game matrix $\mathcal{A}$. Choosing the tactic scouts costs the Blue player X and $\mathcal{P}_t = (\mathcal{R}, \mathcal{J}_t)$ where $\mathcal{J}_t = \mathcal{A}^t$. Choosing the strength scouts costs the Blue player Y and $\mathcal{P}_s = (\mathcal{R}, \mathcal{J}_s)$ where $\mathcal{J}_s = \mathcal{A}^s$. Due to the properties of the Lanchester Equations, we cannot simply subtract the penalty cost from $\mathcal{A}$. The cost for gaining information, troop strength X or Y, must be subtracted from the overall troop strength, B, in the equation for payoffs, $DOLR$. The penalty for gaining information formulates the following payoff equation to define $\mathcal{A}^t$ when a tactic scout is used.

$$DOLR = \begin{array}{l} \left(\frac{c_{bx} * K_{bT_b 1} * [(B-X) * S_b * 0.01]}{R * S_r * 0.01} + \frac{c_{bx} * K_{bT_b 2} * [(B-X) * (100 - S_b) * 0.01]}{R * (100 - S_r) * 0.01} \right) \\ - \left(\frac{K_{rT_r 1} * (R * S_r * 0.01)}{(B-X) * S_b * 0.01} + \frac{K_{rT_r 2} * [R * (100 - S_r) * 0.01]}{(B-X) * (100 - S_b) * 0.01} \right) \end{array}$$

The penalty for sending strength scouts formulates the following payoff equation to define $\mathcal{A}^s$ when a strength scout is used.

$$DOLR = \begin{array}{l} \left(\frac{c_{by} * K_{bT_b 1} * [(B-Y) * S_b * 0.01]}{R * S_r * 0.01} + \frac{c_{by} * K_{bT_b 2} * [(B-Y) * (100 - S_b) * 0.01]}{R * (100 - S_r) * 0.01} \right) \\ - \left(\frac{K_{rT_r 1} * (R * S_r * 0.01)}{(B-Y) * S_b * 0.01} + \frac{K_{rT_r 2} * [R * (100 - S_r) * 0.01]}{(B-Y) * (100 - S_b) * 0.01} \right) \end{array}$$

Apart from the benefit of gaining the increase in efficiency, c_{bx}, by choosing to use tactic scouts, the Blue player also learns which tactic the Red player is using, which reduces her available strategies from 18 to 9 defining $\mathcal{J}_t = (\mathcal{J}_1^t, \mathcal{J}_2^t)$. The reduction in the number of available strategies produces 2 (18 x 9) sub-games located on worksheets "Tactic Scout (1)" (Figure 3) and "Tactic Scout (2)" of the Excel file model. Similarly, by choosing to use strength scouts, the Blue player learns how the

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	T = Tactic = [Attack = 2; Defend = 1]															
2	S = Percentage of Strength in Sector 1 = 10-90															
3	Information Cost (# of Squads Blue loses from Overall Strength to learn that Red uses T = 1) =														9	
4																
5	Solve				Tactic Scout											
6					Red											
7		T		1	1	1	1	1	1	1	1	1		Row		
8			S	10	20	30	40	50	60	70	80	90	z	Strat		
9		1	10	0.04	-0.38	-0.69	-0.95	-1.18	-1.36	-1.46	-'.34	-0.38	1	0		
10		1	20	0.466	0.04	-0.16	-0.29	-0.38	-0.42	-0.38	-0.15	0.822	1	0		
11	Blue	1	30	0.775	0.241	0.04	-0.07	-0.12	-0.13	-0.07	0.169	1.059	1	0		
12		1	40	1.044	0.376	0.149	0.04	-0.01	-0.01	0.059	0.274	1.053	1	0		
13		1	50	1.283	0.472	0.21	0.091	0.04	0.039	0.101	0.288	0.947	1	0		
14		1	60	1.479	0.525	0.226	0.096	J.043	0.04	0.093	0.251	0.789	1	1		
15		1	70	1.6	0.507	0.176	0.041	0.01	0.01	0.04	0.173	0.595	1	0		
16		1	80	1.537	0.323	-0.02	-0.14	-0.17	-0.15	-0.09	C.04	0.363	1	0		
17		1	90	0.747	-0.51	-0.78	-0.82	-0.75	-0.62	-0.47	-0.26	0.04	1	0		
18		2	10	-0.24	-0.61	-0.91	-1.2	-1.46	-1.7	-1.89	-'.98	-1.65	1	0		
19		2	20	0.063	-0.24	-0.41	-0.55	-0.66	-0.75	-0.79	-0.75	-0.32	1	0		
20		2	30	0.248	-0.09	-0.24	-0.34	-0.4	-0.45	-0.45	-0.37	0.036	1	0		
21		2	40	0.393	-0.01	-0.16	-0.24	-0.29	-0.31	-0.3	-0.21	0.154	1	0		
22		2	50	0.508	0.036	-0.12	-0.2	-0.24	-0.25	-0.23	-0.15	0.172	1	0		
23		2	60	0.58	0.036	-0.13	-0.21	-0.24	-0.24	-0.21	-0.13	0.138	1	0		
24		2	70	0.577	-0.03	-0.21	-0.27	-0.29	-0.28	-0.24	-0.16	0.068	1	0		
25		2	80	0.39	-0.27	-0.43	-0.47	-0.45	-0.41	-0.34	-0.24	-0.04	1	0		
26		2	90	-0.52	-1.15	-1.22	-1.15	-1.03	-0.87	-0.69	-0.49	-0.24	1	0		
27		Prob		1	1	1	1	1	1	1	1	1	0	0.04	Game Value	
28		Col. Strat		0	0	0	0	0	1	0	0	0	0.04		0.04	
29														0.04	Game Value	
30																
31			Col		Row	Const	Const	Const	Const	Const	Const	Const	Const	Const	Const	
32			Min		Max	1	2	3	4	5	6	7	8	9	10	
33	Objective: z		0.04		0.04	-1.44	-0.48	-0.19	-0.06	-0	4E-13	-0.05	-0.21	-0.75	1	
34	Const	1	1.4													

Composite Game / No Scout / Tactic Scout (1) / Tactic Scout (2) / Strength Scout (1)

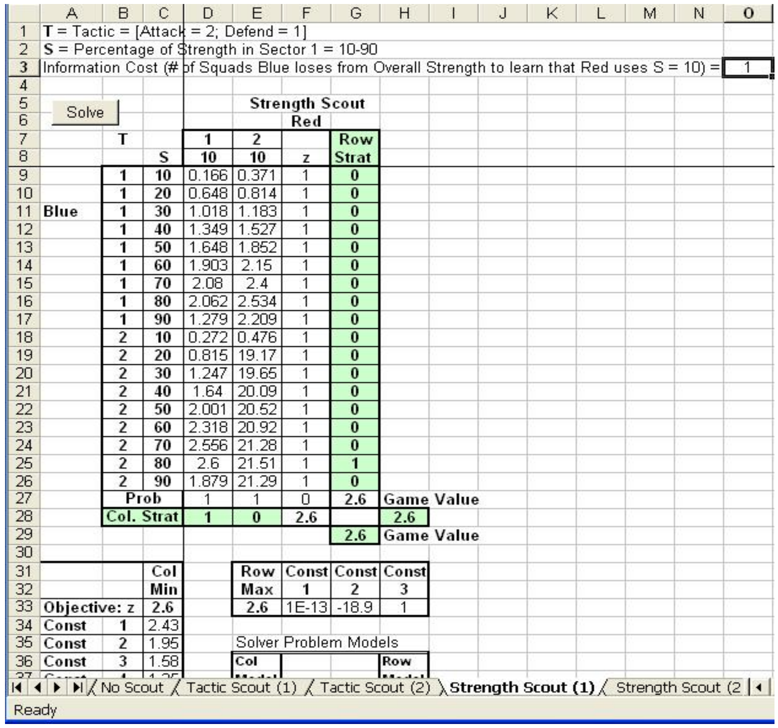
Ready

Figure 3: Excel File Model - Tactic Scout (1) Sub-Game Worksheet

Red player allocates her troop strength, which reduces her available strategies from 18 to 2 defining $\mathcal{J}_s = (\mathcal{J}_{10}^s, \mathcal{J}_{20}^s, \dots \mathcal{J}_{90}^s)$. This reduction in the number of available strategies produces 9 (18 x 2) sub-games located on worksheets "Strength Scout (1)" (Figure 4) through "Strength Scout (9)" of the Excel file model.

In Section 3, we only mentioned a penalty attached to using one of the scouts. If this were the case, one would expect $a_{ij} \geqslant a_{ij}^j$ where $a_{ij} \in \mathcal{A}$ and $a_{ij}^j \in \mathcal{A}^j$. However, due to using the Mixed Law of the Lanchester equations, $a_{ij} \leqslant a_{ij}^j$ often is the case in our model. It is important to point out that the increase in efficiency, c_b, can outweigh the penalty. During the analysis of the results, we must consider whether the information gain due to the increase in efficiency causes the player to use a scout or whether the information gain due to the partitioning causes the player to use a scout.

It generally appears that the information that provides more partitions and more sub-games is the most valuable information. In order to determine the value of the information that scouts



	A	B	C	D	E	F	G	H
1	T = Tactic = [Attack = 2; Defend = 1]							
2	S = Percentage of Strength in Sector 1 = 10-90							
3	Information Cost (# of Squads Blue loses from Overall Strength to learn that Red uses S = 10) = 1							
5	Solve			Strength Scout				
6				Red				
7		T		1	2		Row	
8			S	10	10	z	Strat	
9		1	10	0.166	0.371	1	0	
10		1	20	0.648	0.814	1	0	
11	Blue	1	30	1.018	1.183	1	0	
12		1	40	1.349	1.527	1	0	
13		1	50	1.648	1.852	1	0	
14		1	60	1.903	2.15	1	0	
15		1	70	2.08	2.4	1	0	
16		1	80	2.062	2.534	1	0	
17		1	90	1.279	2.209	1	0	
18		2	10	0.272	0.476	1	0	
19		2	20	0.815	19.17	1	0	
20		2	30	1.247	19.65	1	0	
21		2	40	1.64	20.09	1	0	
22		2	50	2.001	20.52	1	0	
23		2	60	2.318	20.92	1	0	
24		2	70	2.556	21.28	1	0	
25		2	80	2.6	21.51	1	1	
26		2	90	1.879	21.29	1	0	
27		Prob		1	1	0	2.6	Game Value
28		Col. Strat		1	0	2.6		2.6
29							2.6	Game Value
31			Col		Row	Const	Const	Const
32			Min		Max	1	2	3
33	Objective: z		2.6		2.6	1E-13	-18.9	1
34	Const	1	2.43					
35	Const	2	1.95		Solver Problem Models			
36	Const	3	1.58		Col			Row

No Scout / Tactic Scout (1) / Tactic Scout (2) / Strength Scout (1) / Strength Scout (2

Ready

Figure 4: Excel File Model - Strength Scout (1) Sub-Game Worksheet

provide, we assemble the composite game (Figure 5) discussed in Section 4 from the main game and all sub-games. Using the information gained by solving the main game and all of the sub-games, we assemble the composite game to determine which type of scout the Blue player should use and the value of the game. The Blue player can do no worse than the value of the main game because he can simply choose to use no scouts. In this composite game, $\mathcal{A}^p$, the Blue player has three strategies; no scouts, tactic scouts, or strength scouts, while the Red player has the original 18 strategies. The payoffs in this (3 x 18) composite game matrix are the expected payoffs for the Blue player. These expected payoffs are computed by the dot products from the appropriate game or sub-game. For instance, the entry in the composite game matrix at the intersection of no scouts and the Red player defending with 50% in sector 1 is the transpose of the column of the Red player's strategy (1, 50) from the "No Scout" main game, $a_{ij}\,\forall\{i \in \mathcal{R};\, j \in \mathcal{C};\, j = (1,50)\}$, dotted with the optimal pure/mixed strategy of the Blue player from the main game, p^0. Likewise, the entry in the composite game matrix at the intersection of tactic scouts and the Red player attacking with 60% in sector 1 is the transpose of the column of the Red player's strategy (2,60) from the "Tactic Scout (2)" sub-game, $a_{ij}\,\forall\{i \in \mathcal{R};\, j \in \mathcal{J}^t_2;\, j = (2,60)\}$, dotted with the optimal pure/mixed strategy of the Blue player from the "Tactic Scout (2)" sub-game, $p^{t,2}$.

All of the sub-games and the composite game have the same structural properties as the main game. The result of solving the composite games tells us the probability that the Blue player should

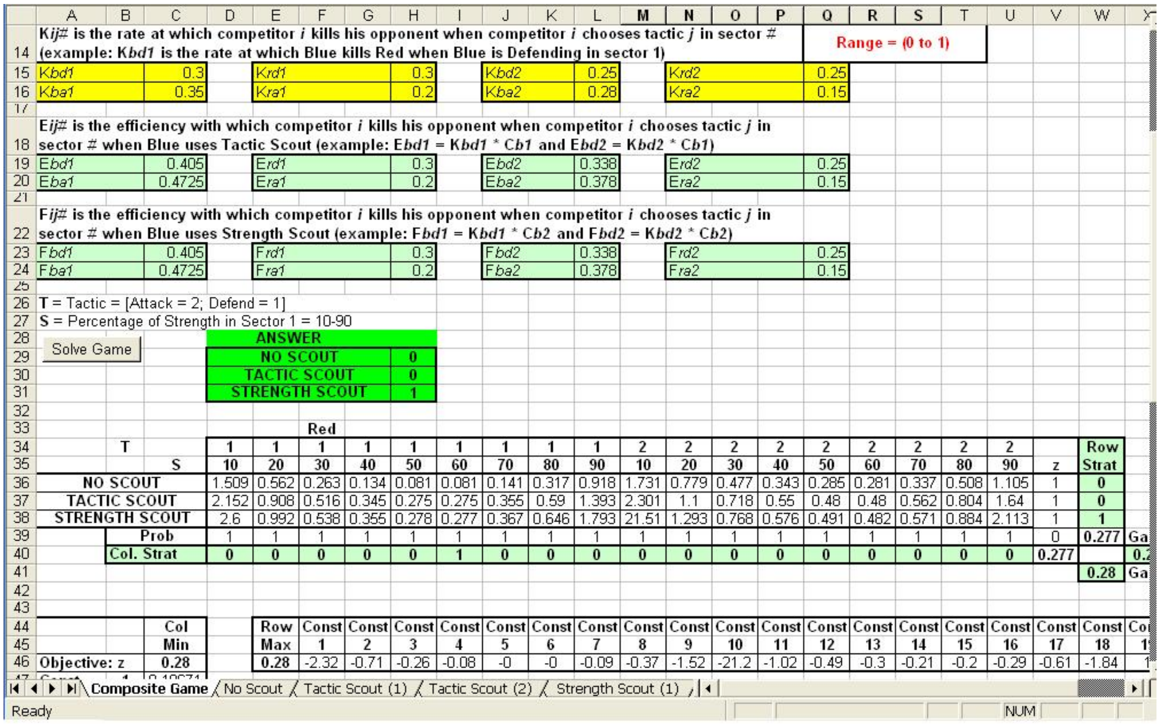
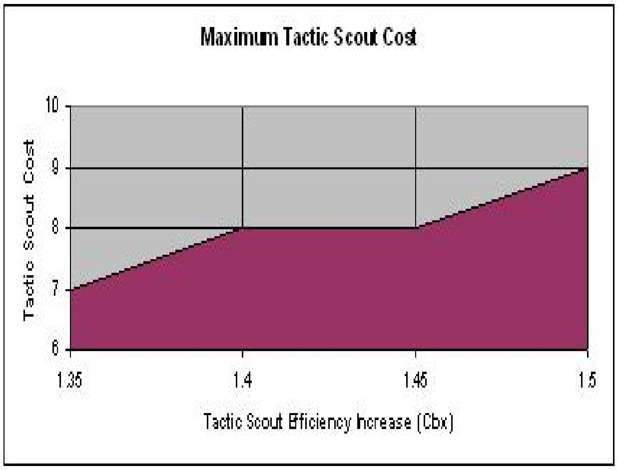
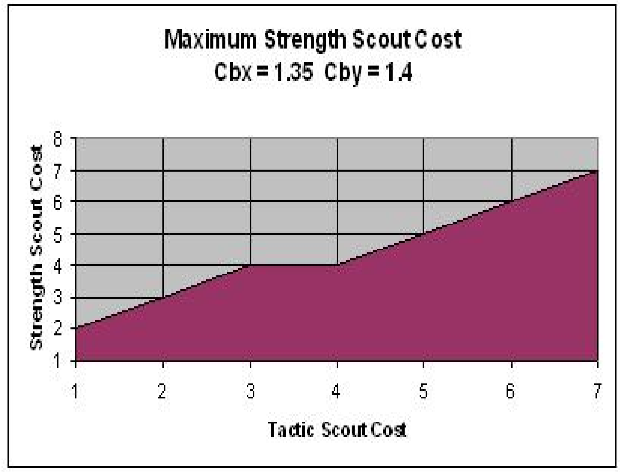


Figure 5: Excel File Model - Composite Game Worksheet

send no scouts, ρ_0, tactic scouts, ρ_1, or strength scouts, ρ_2 (H29 through H31 in Figure 5). If the Blue player should send scouts, the difference in the value of composite game and the "No Scout" game is the game value of the information that the scout gains. As such, the Excel file model is designed for the user to do a parametric analysis on the cost of the scouts (X or Y) and the increase in efficiency (c_b) due to those scouts. Through this type of analysis we can develop a trade-off curve based on the maximum cost or troop strength the player should be willing to sacrifice to gain the information based on increase in efficiency that scout returns (Figure 6, left). This trade-off curve forms a feasibility region in which the scouts should be used if the information increases efficiency appropriately. This maximum cost feasibility region is the usable value of information that the player can use. For instance, if the player believes he can reasonably gain the information by sacrificing a troop strength level within the feasibility region, then the sacrifice is of value to him to gain that information.

Within the intersection of feasibility regions of multiple scout types, there is another trade-off curve that tells the player which type of scout or type of information is more valuable given a certain increase in efficiency due to that information (Figure 6, right). We explain the results from the model and the analysis of the value of information in the next section.





Feasibility Region
(Tactic Scout feasible in red or darker area)

Intersection of Feasibility
(Strength Scout = red; Tactic Scout = gray)

Figure 6: Example Trade-Off Curves and Feasibility Regions

6 Results and Analysis

In order to test the model, we developed five scenarios to analyze how the model behaved with different inputs and to determine the value of the information in each scenario. The cost of the scouts, X and Y, is what we are trying to determine. Remember, the usable value of information for the player is the maximum cost that he should be willing to sacrifice to gain the information at a given increase in efficiency level. For all five scenarios, we tested the same range for the increase in efficiency due to information, c_{bx} and c_{by}. We analyzed situations where $c_{bx} \in (1.35, 1.4, 1.45, 1.5)$ and $c_{by} \in (1.35, 1.4, 1.45, 1.5)$ to form the feasibility regions. The idea is that gaining this information will increase the Blue player's efficiency anywhere from 35% to 50%. Based on his feelings on this increase in efficiency, he can determine the value of this information using the boundary of the feasibility region. We defined the five scenarios as shown in Table 2 and described them below.

Table 2: Scenarios

Scenario	B	R	K_{bd1}	K_{bd2}	K_{rd1}	K_{rd2}	K_{ba1}	K_{ba2}	K_{ra1}	K_{ra2}
1	40	50	0.35	0.3	0.2	0.15	0.2	0.15	0.15	0.1
2	40	50	0.4	0.2	0.22	0.1	0.3	0.15	0.15	0.08
3	40	50	0.3	0.2	0.18	0.1	0.35	0.15	0.2	0.08
4	50	50	0.3	0.25	0.3	0.25	0.2	0.15	0.2	0.15
5	50	50	0.3	0.25	0.3	0.25	0.35	0.28	0.2	0.15

In scenarios 1 through 3, the Blue player is somewhat undermanned. The main purpose of these scenarios is to show that even when Blue has a troop strength deficit, he is still willing to make the deficit greater by sacrificing troop strength to gain information. Scenario 1 represents a situation in which both players are better prepared to defend in each sector than attack. Sector 1 is slightly more favorable for a defense and attack by both players. Although the Blue player has a troop strength deficit, he is either better equipped and/or better trained and can kill the Red troops more efficiently, representing asymmetry on the battlefield.

Scenario 2 also represents a situation where both players are better prepared to defend in each sector than attack, as this is the typical case. However, Sector 1 is drastically more favorable than sector 2 for a defense and attack by both players. Again, the Blue player is either better equipped and/or better trained.

In scenario 3, we designed a situation where the Blue player is better prepared to attack in sector 1 and defend in sector 2. The Red player is equally prepared to attack or defend in sector 1 and better prepared to defend in sector 2. As before, the Blue player is either better equipped and/or better trained.

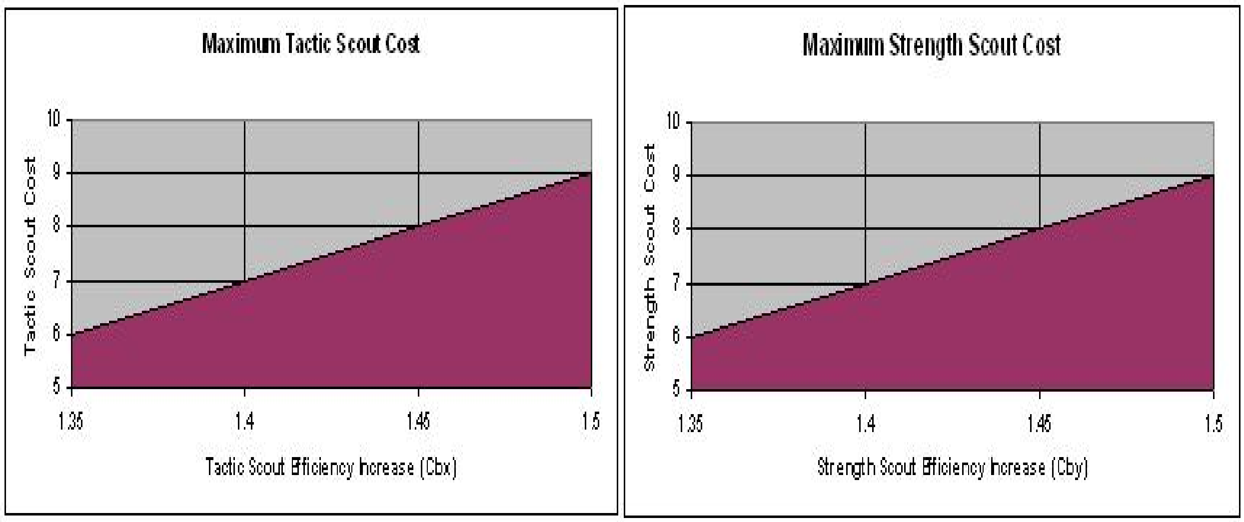
At the request of ARO, scenario 4 is a very symmetric situation in which both players have the same troop strength, sector 1 is slightly more favorable for a defense and attack by both players, and both player are equally equipped and/or trained.

The purpose of the symmetry in scenario 4 is to demonstrate and analyze the effect of increasing the Blue player's capability to attack in scenario 5. Scenario 5 represents a situation where the Blue player receives an additional capability, such as aircraft arriving in theater, allowing him to attack more effectively than he could before, as in scenario 4.

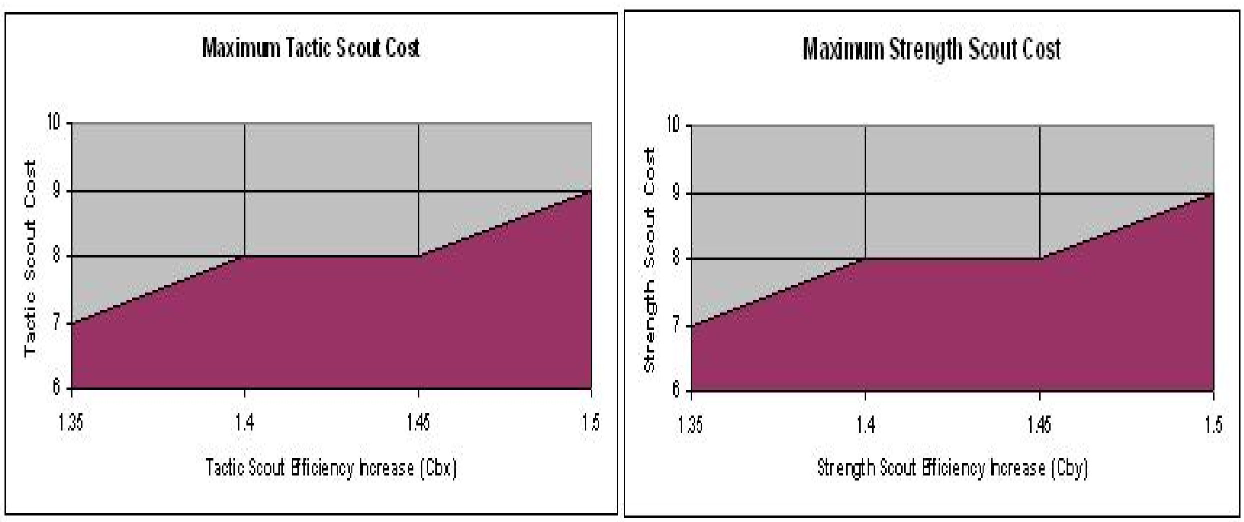
The feasibility region graphs and the intersection of feasibility region graphs for all scenarios are located in Appendix C - Graphs. Here, we draw particular attention to the graphs for scenarios 4 and 5. By comparing these two scenarios, we show that the feasibility regions (Figure 7), as in all scenarios, are shaped by the Lanchester equation variables. As seen in scenario 5, the increase in attack capability increases the feasibility region when c_{bx} and c_{by} each equal 1.4. This increased capability also changes the shape of the trade-off curve in the intersection of feasibility regions graphs (Figure 8).

Similar trends are evident throughout Appendix C - Graphs. As the variables in the Lanchester equations increase for the Blue player as compared to the Red player, the Blue player's value of information increases.

These results may lead one to believe that the somewhat arbitrarily assigned variables in the Lanchester equations, especially the increase in efficiency variables, are the only pieces of the model



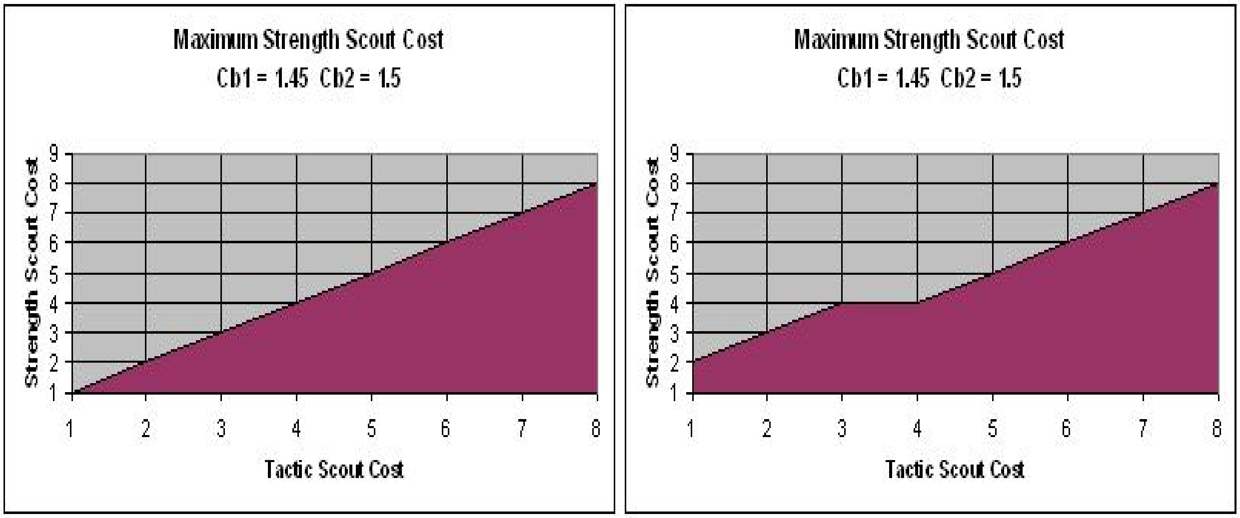
Scenario 4 Feasibility Region (Scout feasible in red or darker area)



Scenario 5 Feasibility Region (Scout feasible in red or darker area)

Figure 7: Scenario 4 & 5 Feasibility Regions

that affect the value of information and the theory of partitioning has no effect. However, in all instances in the feasibility regions where $c_{bx} = c_{by}$ and $X = Y$, the Strength Scout sub-game always has a higher game value than the Tactic Scout sub-game and No Scout game. In scenario 5, the Strength Scout's game value is as much as 0.003 greater than the Tactic Scout's game value. Furthermore, in scenario 3, we set $c_{bx} = c_{by} = 1$ meaning there is no increase in efficiency due to the gain of information and set the cost of scouts very low, $X = Y = 0.001$. This scenario resulted in the use of the Strength Scout with a game value 0.00058 greater than the No Scout game value. Since there is no increase in efficiency, this increase in payoff is due solely to partial knowledge of the opponent's strategy. Although the value of this kind of knowledge is fairly small



Scenario 4 (left); Scenario 5 (right)
(Strength Scout used in red or darker area; Tactic Scout used in gray or lighter area)

Figure 8: Scenario 4 & 5 Intersection of Feasibility Regions

in the context of this game, it is probably due to the simplicity of strategies – they are almost always pure strategy solutions.

Our analysis also shows that in this model using Lanchester equations, the K values have the greatest effect on optimal strategies. When the K values were somewhat equal or even slightly more favorable for one sector over another as in scenarios 1, 4 and 5, both players almost always allocated there troop strength equally to both sectors. However, when the K values highly favor one of the sectors over another as in scenarios 2 and 3, both players always send a greater troop strength to the more favorable sector as expected. Likewise, when one tactic dominates with respect to K values, both players select their most dominant tactic.

The Strength Scout sub-games and the Composite game always resulted in pure optimal strategies. We believe that these game spaces are to small and the payoffs are too varied to result in mixed optimal strategies. The No Scout game resulted in mixed optimal strategies for scenarios 1, 3, and 5 and pure optimal strategies for scenarios 2 and 4. The Tactic Scout sub-games almost always resulted in mixed optimal strategies for all scenarios except 2, which always produced a pure optimal strategy.

All of the strategies found in all scenarios resulted in a pure strategy choice of which (if any) scout to use. It may well be that there are no mixed scout choice strategies for any choice of game parameters, but in any case the user can rely on a pure scout choice strategy with a fair degree of certainty.

7 Conclusion

We explored several concepts through the development of a value for information including game theory, linear programming, and the Lanchester equations. The use of Lanchester equations provided a theoretical base for modeling attrition in the competitive game of war. In some cases the equations cloud the work with variables that are assigned arbitrarily in their own rite. By exploring and understanding the effects of these variables on the results using multiple scenarios, we developed a useful definition for the value of information instead of a conceptual game value. Most importantly, through careful analysis, we proved our hypothesis. We demonstrated that there is value in the information gained by partitioning based on the information set and more partitions or information sets provide more valuable information.

8 Appendix A - Loading the Model

We highly recommend that the user not alter any cells, formulas, or macros other than the cells highlighted yellow on the top of the "Composite Game" worksheet. Changes to these cells are described in Appendix B - Using the Model. In order to use the model correctly, the user will need to enable macros for this file, load the Solver Add-In, and add the reference to the SOLVER.XLA. All procedures are described below.

8.1 Enabling Macros

The model contains several macros that link the Solve buttons to other macros that automatically solve numerous linear programs simultaneously to return the correct answer. The user will need to enable macros for this model to work correctly. In order to enable macros, the user may need to change his security levels in Excel. Under the Tools menu in Excel, select Macro submenu and the Security command. This process will open a Security window. On the Security Level tab, select Medium. It is necessary to close and restart Excel for this change to take effect. We recommend loading the Solver Add-In before doing so. If you have difficulties enabling macros, simply enter "enable macros" in Excel help.

8.2 Loading the Solver Add-in

The Solver Add-In is the tool Excel uses to solve the linear programs in the macros. This tool is not normally loaded during installation of Excel, but the process to load it is very easy. To check to see if it is loaded on your system, select the Tools menu in Excel and expand the entire menu if necessary. Solver will be a command located in this menu just above the Macro submenu if it is loaded. If it is not loaded, on the Tools menu, click Add-Ins to open a window labeled Add-Ins. In this new window, scroll down to find and check the box to the left of Solver Add-In, and then click the Ok button. If you get a message asking whether you want to install this feature, select Yes. Loading this tool may require the Microsoft Office installation disk and/or administrator rights on your system. If you have difficulties loading this tool, simply enter "solver add-in" in Excel help.

8.3 Map Reference to SOLVER.XLA

The macros for this model are managed through Microsoft's Visual Basic Editor. A common problem in this package is that the macro cannot locate SOLVER.XLA even though the add-in is loaded. The user must locate and map this reference for the macro to work properly. This process is a little more difficult than the previous two processes. Before starting this process, it will be beneficial to locate the SOLVER.XLA file on your computer using the search function on the start menu. A typical path for this file is C:\\Program Files\Microsoft Office\Office\Library\Solver\SOLVER.XLA. With the model now open in the Excel window, the user must first open the Microsoft Visual Basic window by opening the Tools menu, selecting the Macro submenu and the Visual Basic Editor command. Now operating in the Microsoft Visual Basic window, select the Tools menu and the References command. In the new dialogue box, scroll down and select SOLVER. With SOLVER highlighted, click the Browse button, change the file type to All Files or Microsoft Excel Files, navigate the path to SOLVER.XLA on your computer and select Open. With the path properly mapped, the user must save the file, close all Excel windows, restart Excel, and reload the file. If you have difficulties following these instructions or mapping the reference, search for "reference SOLVER.XLA" on the Internet. This problem is common enough that there are multiple links explaining this process through several different techniques.

9 Appendix B - Using the Model

Once the model is loaded correctly, the model is very easy to use. The user only needs to make inputs or change the 14 variables on the first worksheet labeled "Composite Game." The variables described in Table 1 are all highlighted yellow in rows 4 through 19. The model calculated all payoffs for all games and sub-games using these 14 variables. The range for these variables is located in red text near the description of each variable in the model. Please do not change any other cells or formulas.

The light green highlighted cells are secondary answers from the model. The light green rows 19 and 20 on the "Composite Game" page are the product of c_{bx} and K_b for use in the Tactic Scout sub-games. The light green rows 23 and 24 on the "Composite Game" page are the product of c_{by} and K_b for use in the Strength Scout sub-games. The light green right-most column and bottom-most row of each game and sub-game matrix represent p and q respectively. The light green cells on the bottom left of each matrix are the game values for that game. The bright green cells in rows 28 through 31 are the primary answers, rho, that tell the user which scouts are used.

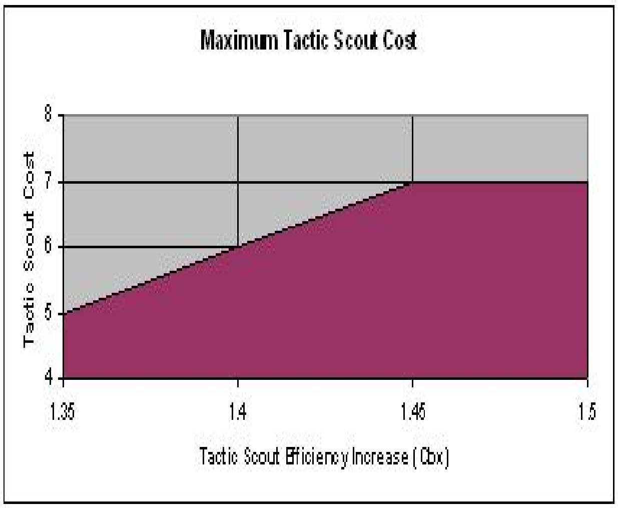
In order to determine these answers, the user simply clicks the "Solve Game" button in the vicinity of cell A28 once the 14 variables are entered appropriately. This button starts a series of macros that solve numerous linear programs simultaneously. This process takes about 10 to 15 seconds.

Once the model returns a solution, the user can trace the appropriate strategies using the primary and secondary answers if the primary answers, rho, are a pure strategy. The primary answer tells the user which game or sub-game to trace. If the pure optimal strategy is no scout, the user should select the "No Scout" worksheet and look at the light green secondary answers on that worksheet for the optimal strategies. If the pure optimal strategy is tactic scout, the user should select the "Tactic Scout (T_r)" worksheet where T_r is determined using rows 34 and 40 on the "Composite Game" worksheet. On the appropriate worksheet, the user should look at the light green secondary answers on that worksheet for the optimal strategies, p and q. If the pure optimal strategy is strength scout, the user should select the "Strength Scout ($\frac{S_r}{10}$)" worksheet where S_r is determined using rows 35 and 40 on the "Composite Game" worksheet. On the appropriate worksheet, the user should look at the light green secondary answers on that worksheet for the optimal strategies, p and q.

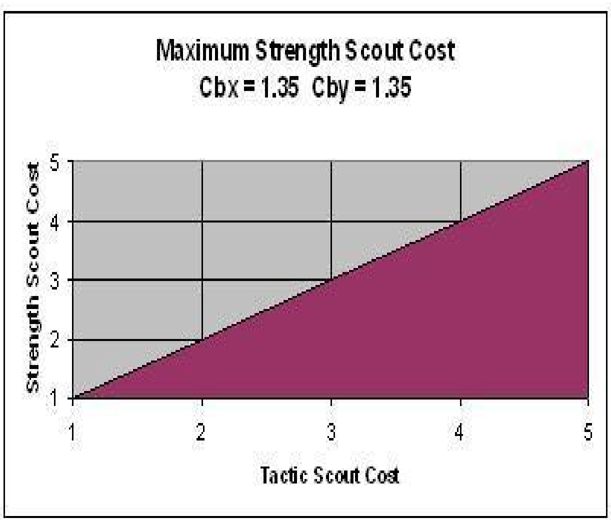
10 Appendix C - Graphs

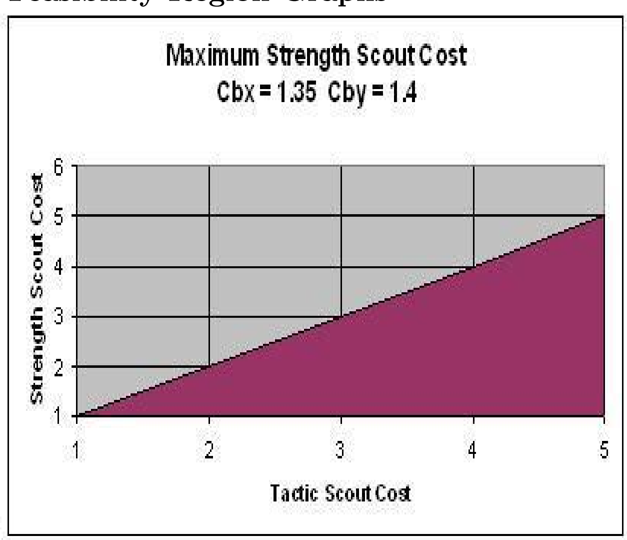
For the Feasibility Region graphs, the user should choose to use a scout any time the trade-off between Cost and Increase in Efficiency falls in the red or darker shaded area. For the Intersection of Feasibility Region graphs, on the graph for the appropriate Increase in Efficiency Levels, the user should choose to use Strength Scouts any time the trade off between the Cost of Tactic Scouts and Cost of Strength Scouts falls in the red or darker area and use Tactic Scouts in the gray or lighter area.

Scenario 1 Feasibility Region Graphs

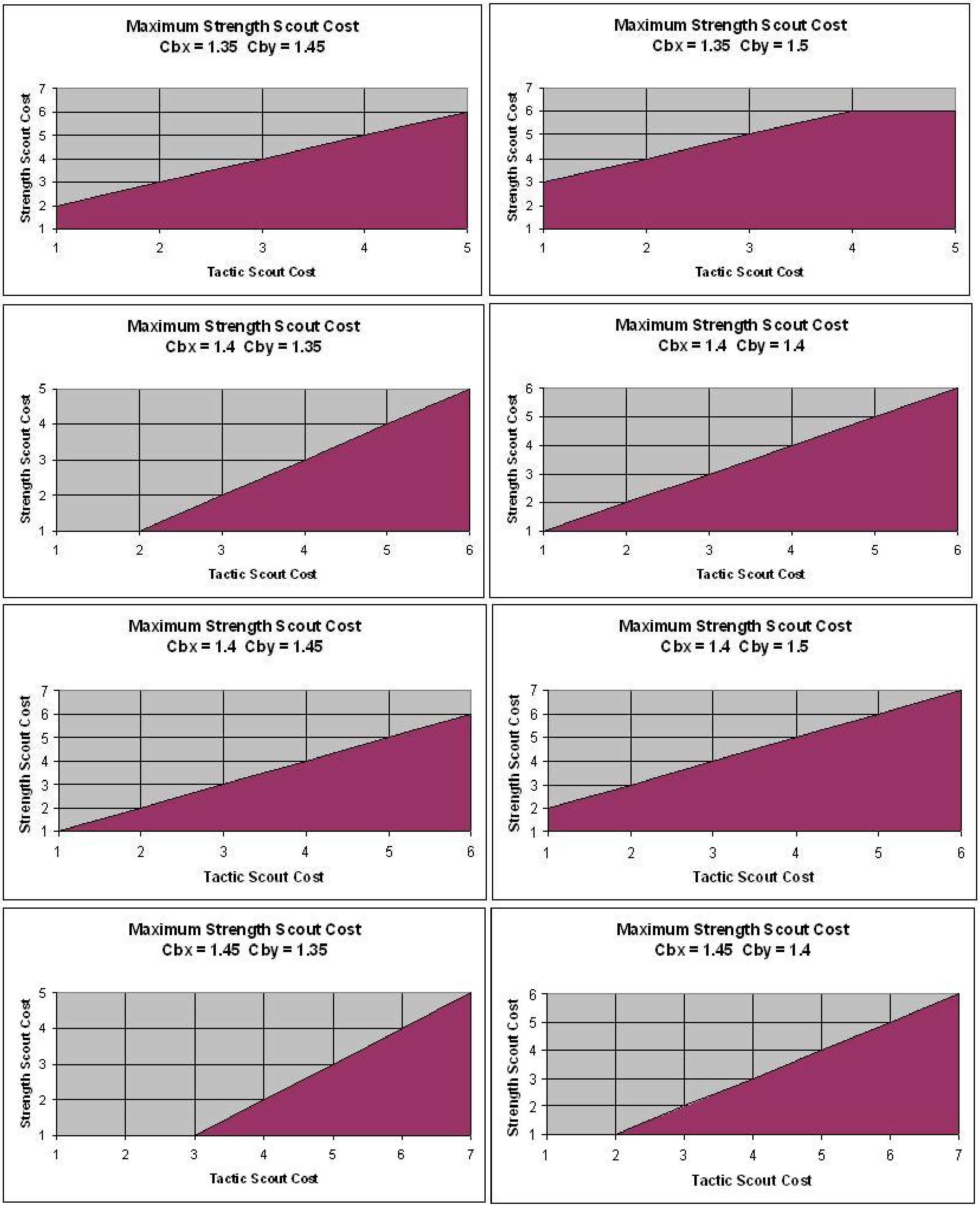


Scenario 1 Intersection of Feasibility Region Graphs

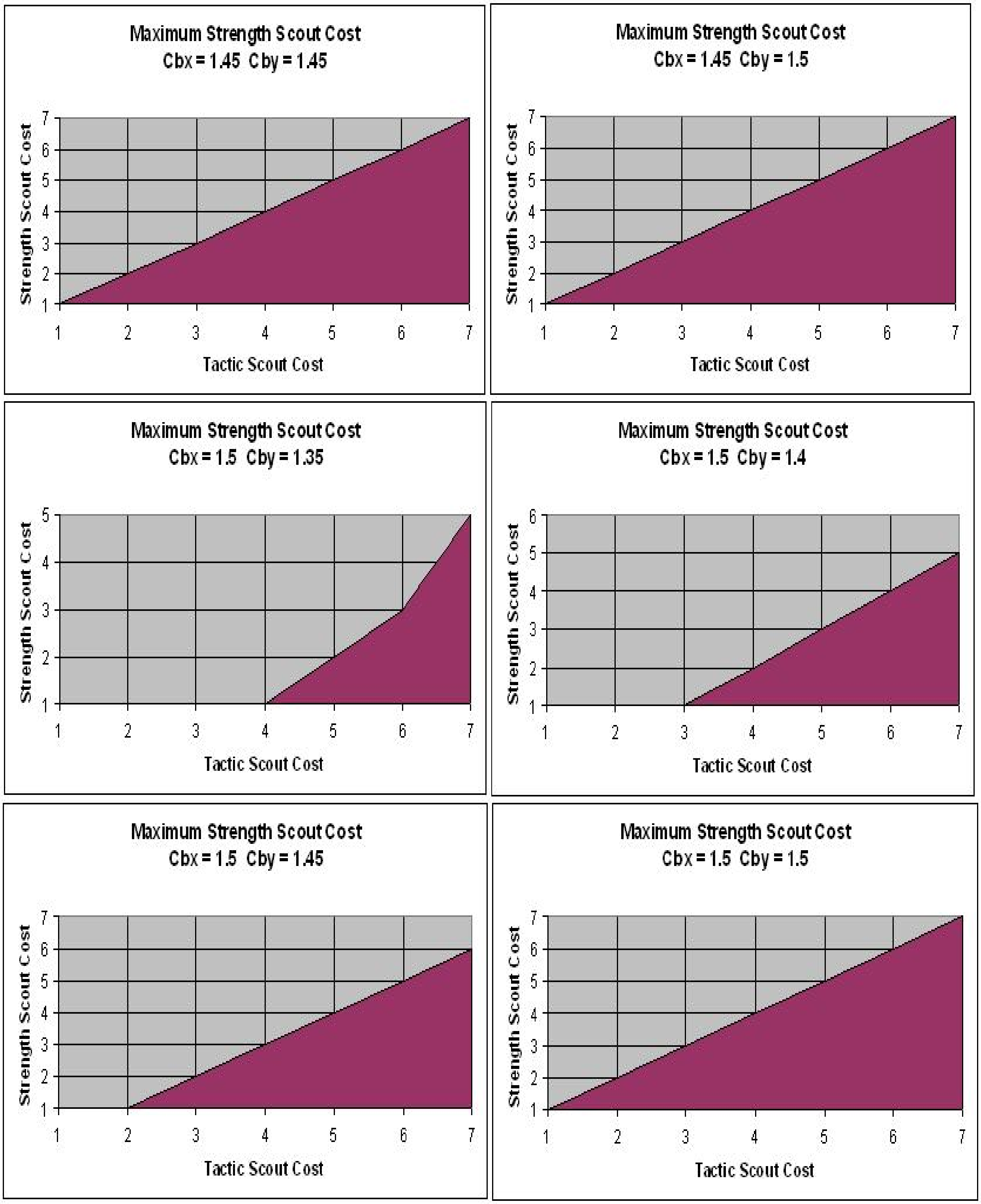




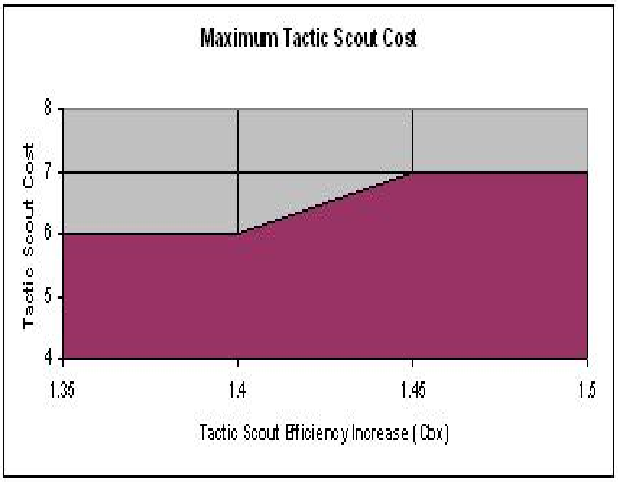
Scenario 1 Intersection of Feasibility Region Graphs



Scenario 1 Intersection of Feasibility Region Graphs



Scenario 2 Feasibility Region Graphs



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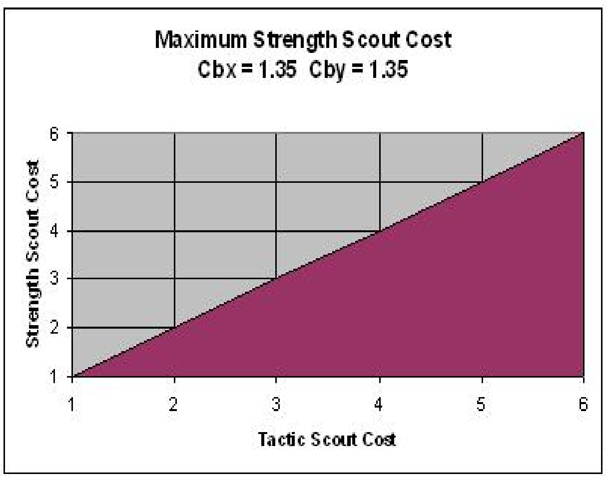
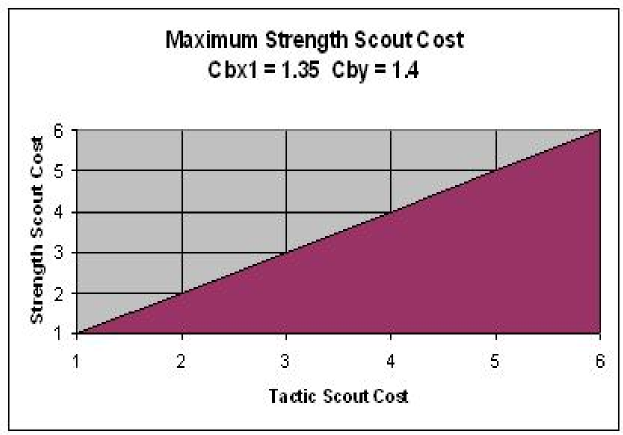
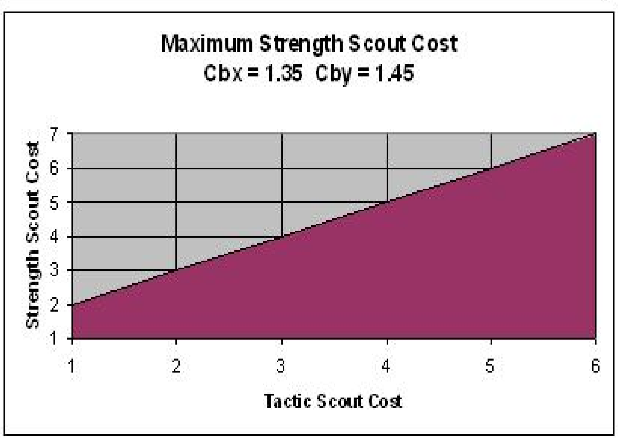
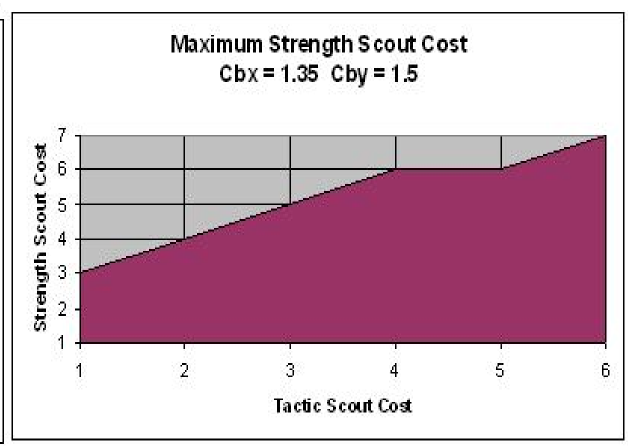
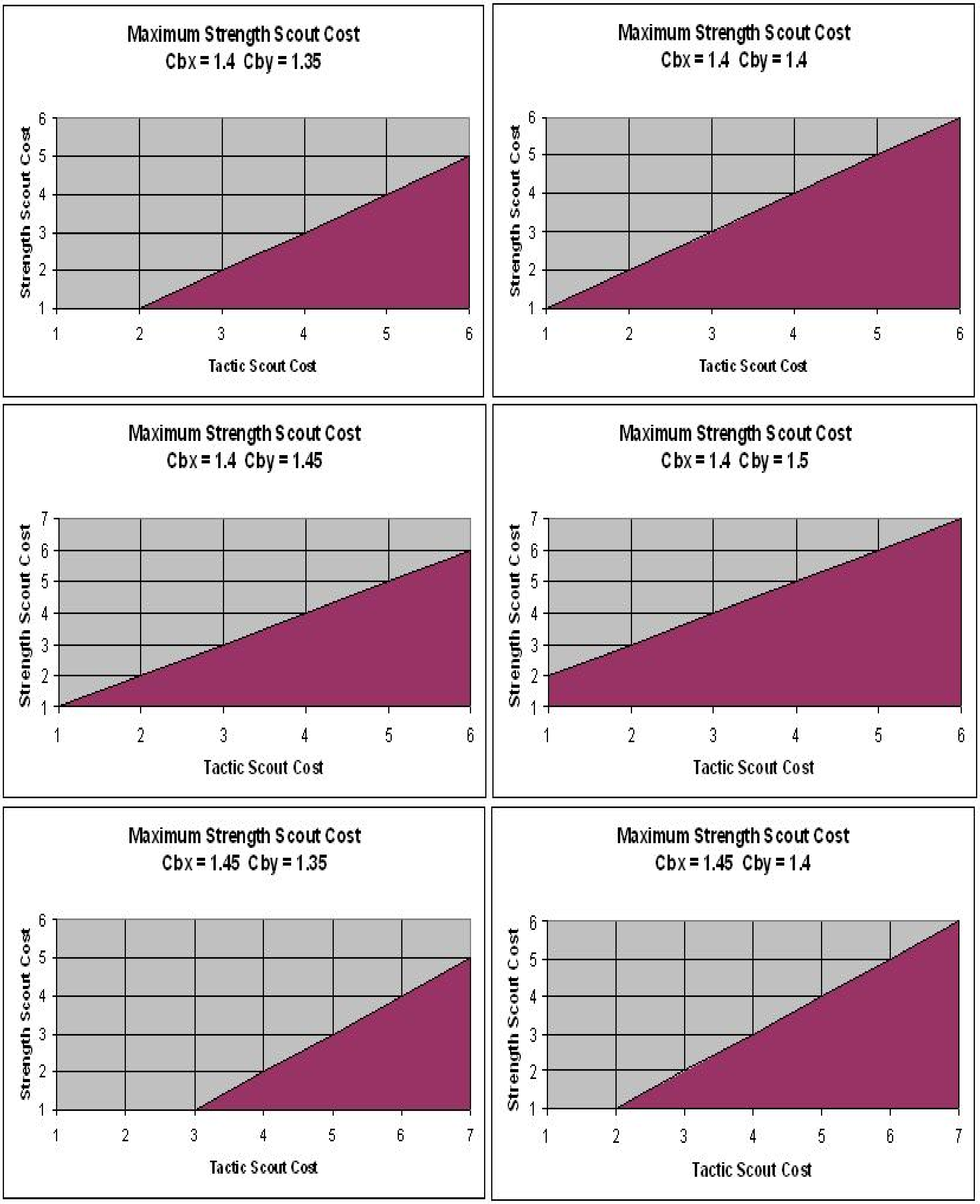




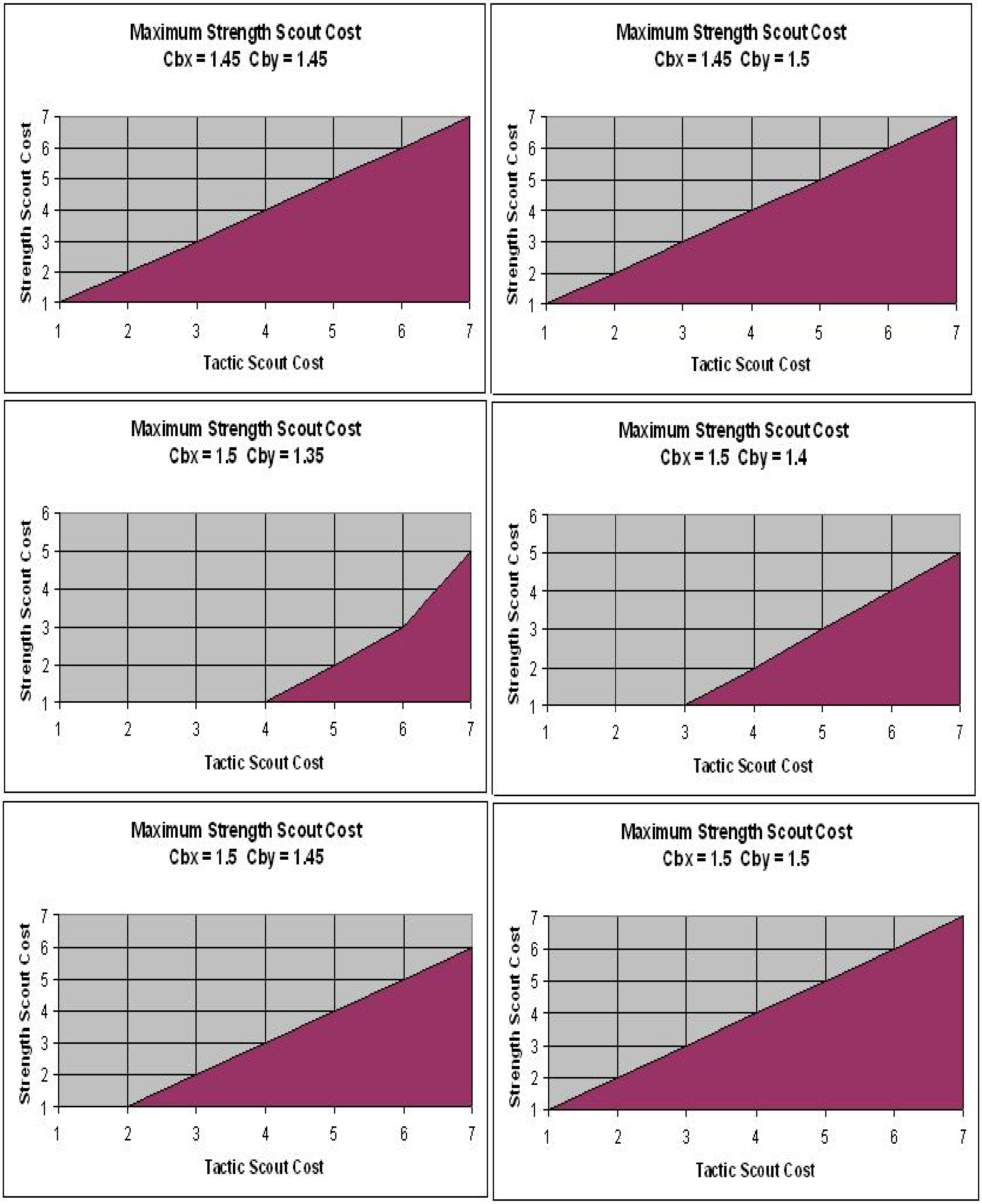




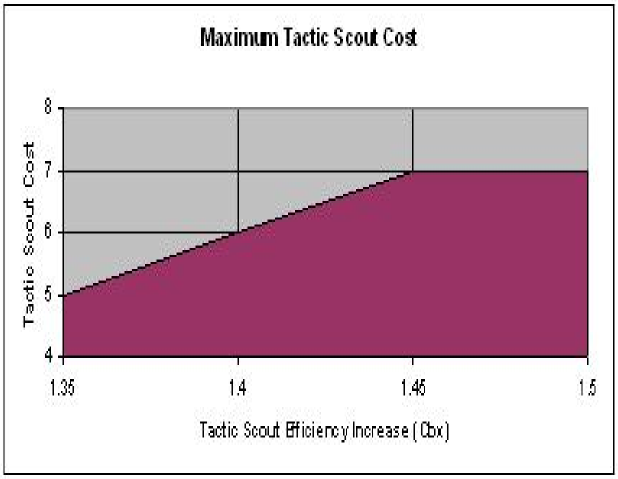
Scenario 2 Intersection of Feasibility Region Graphs

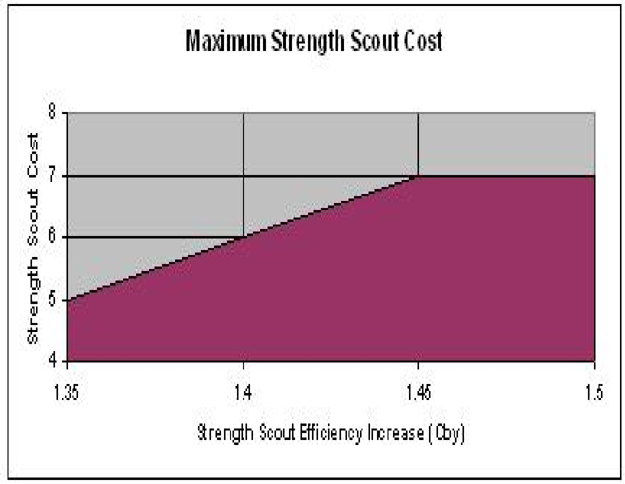


Scenario 2 Intersection of Feasibility Region Graphs

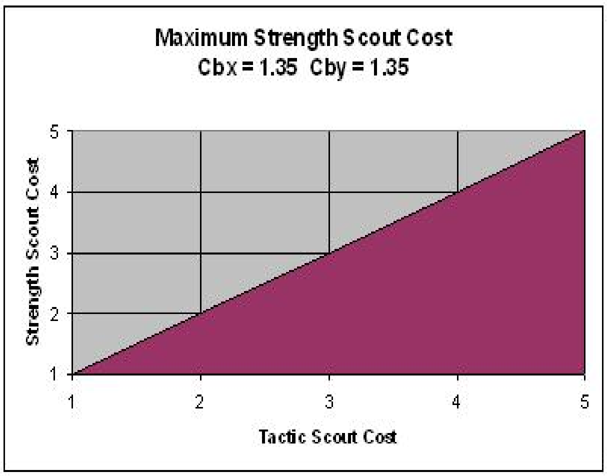


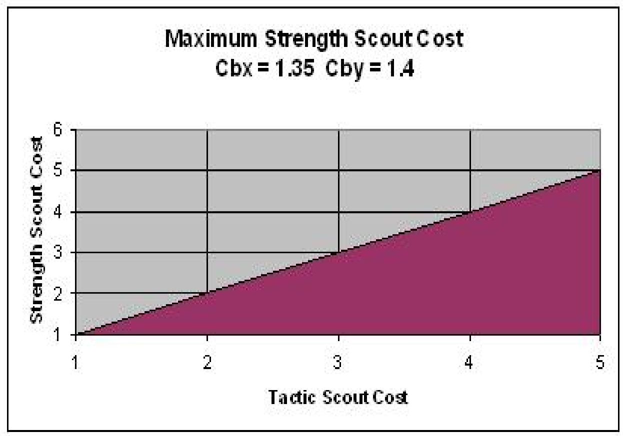
Scenario 3 Feasibility Region Graphs

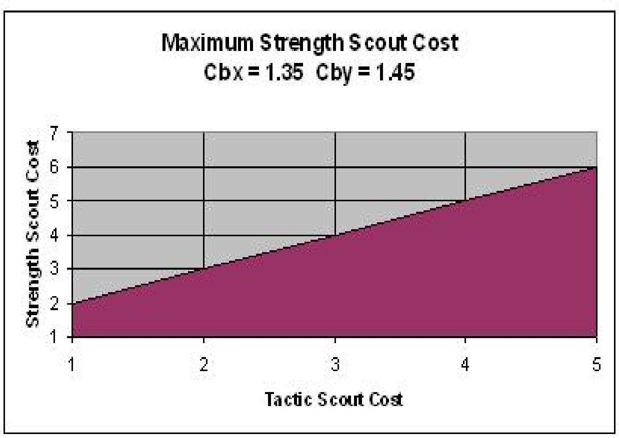


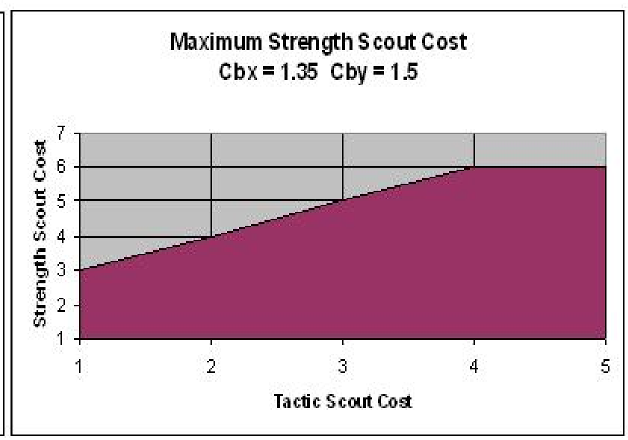


Scenario 3 Intersection of Feasibility Region Graphs

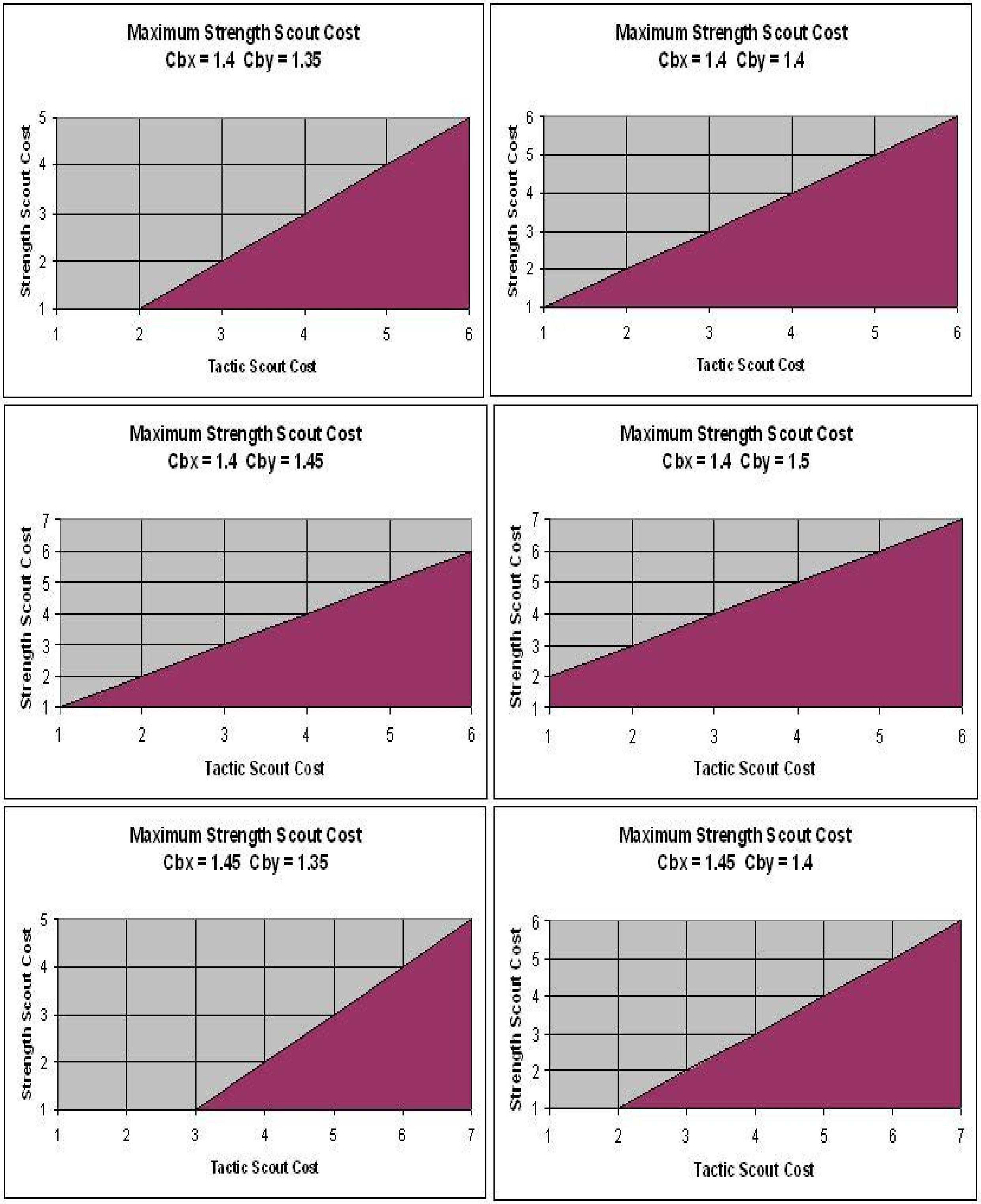








Scenario 3 Intersection of Feasibility Region Graphs



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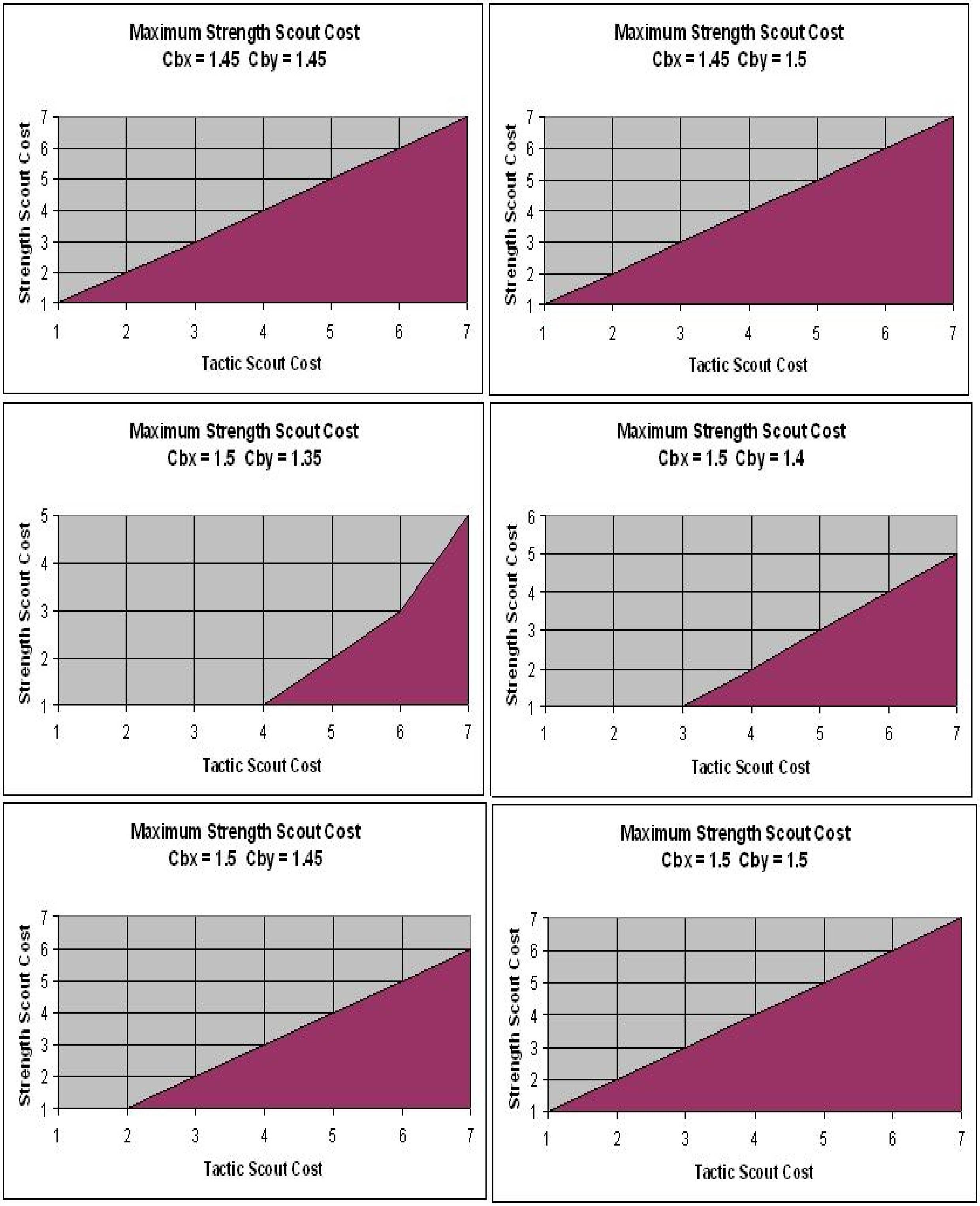
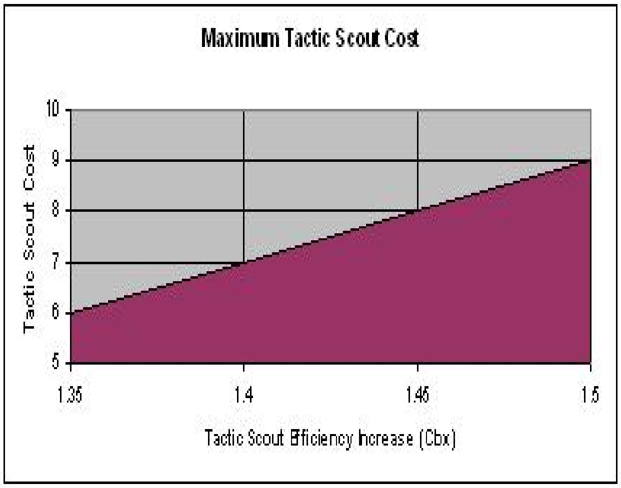
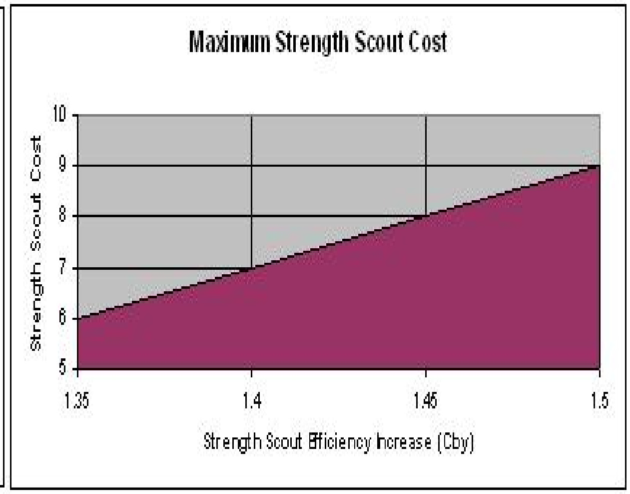


Scenario 4 Feasibility Region Graphs





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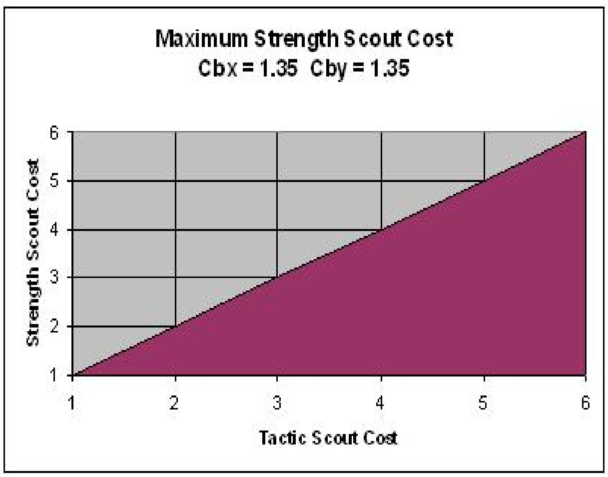

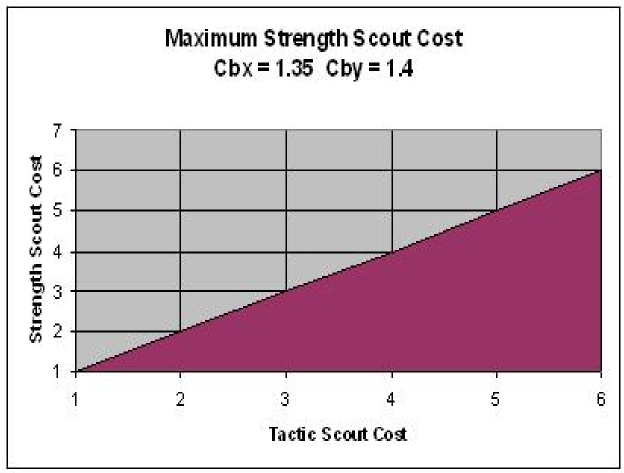

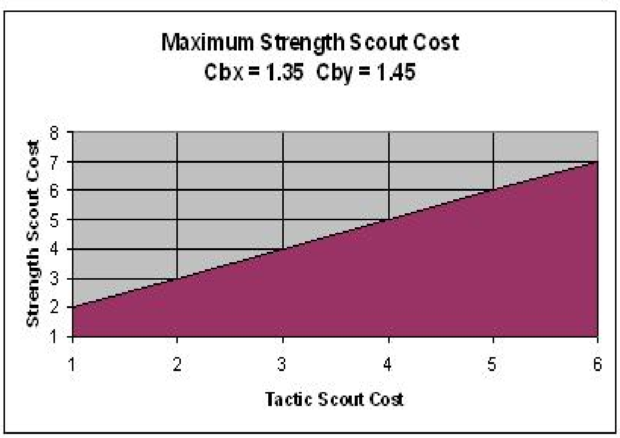

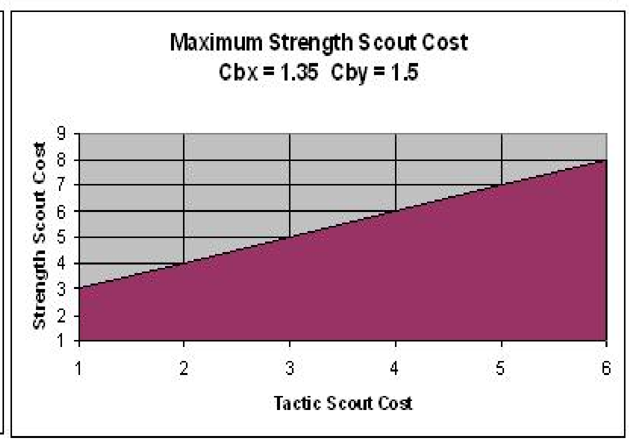

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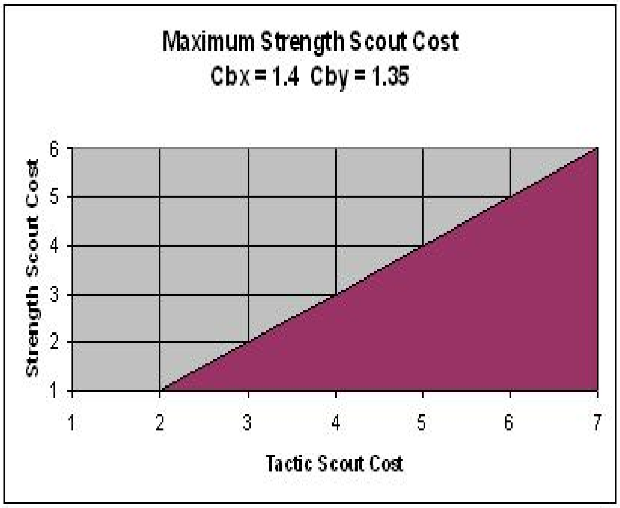

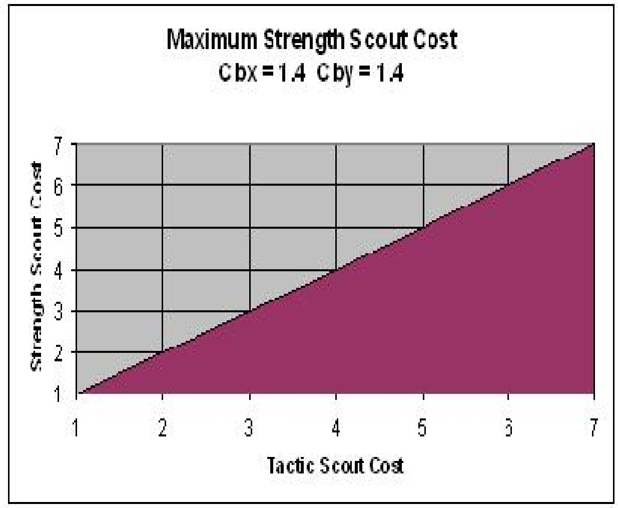

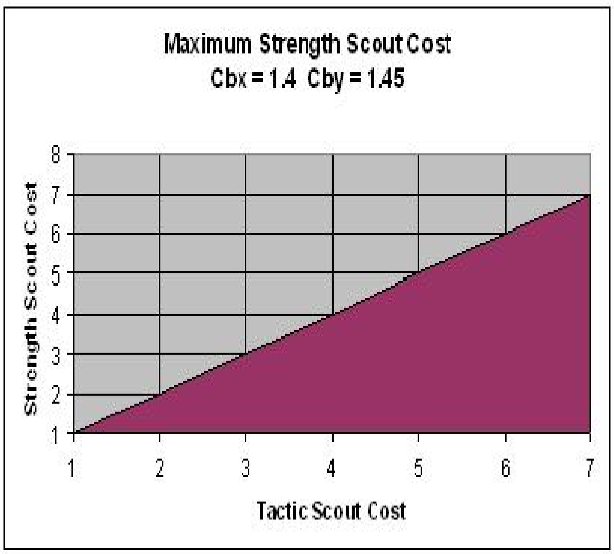

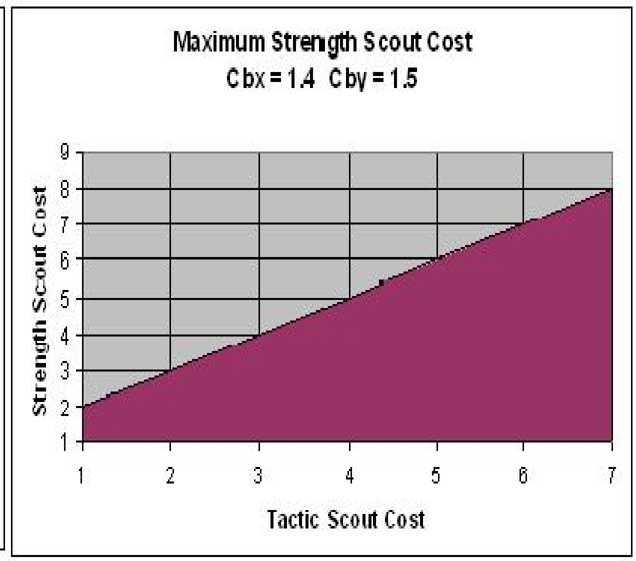

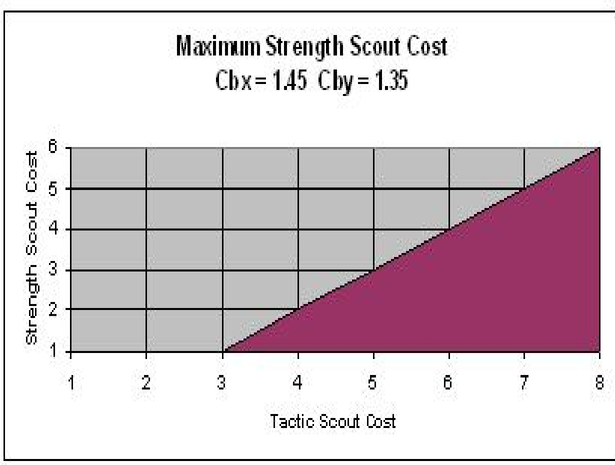

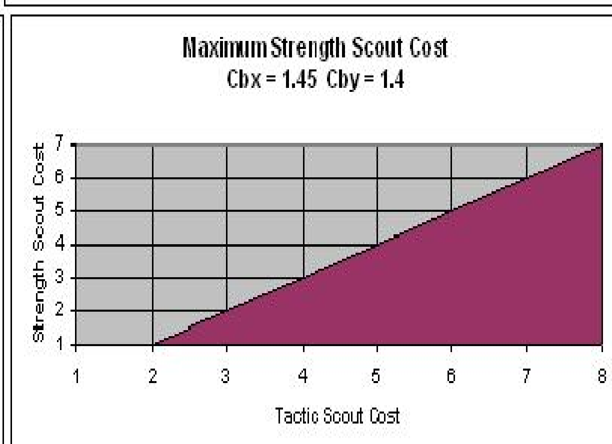

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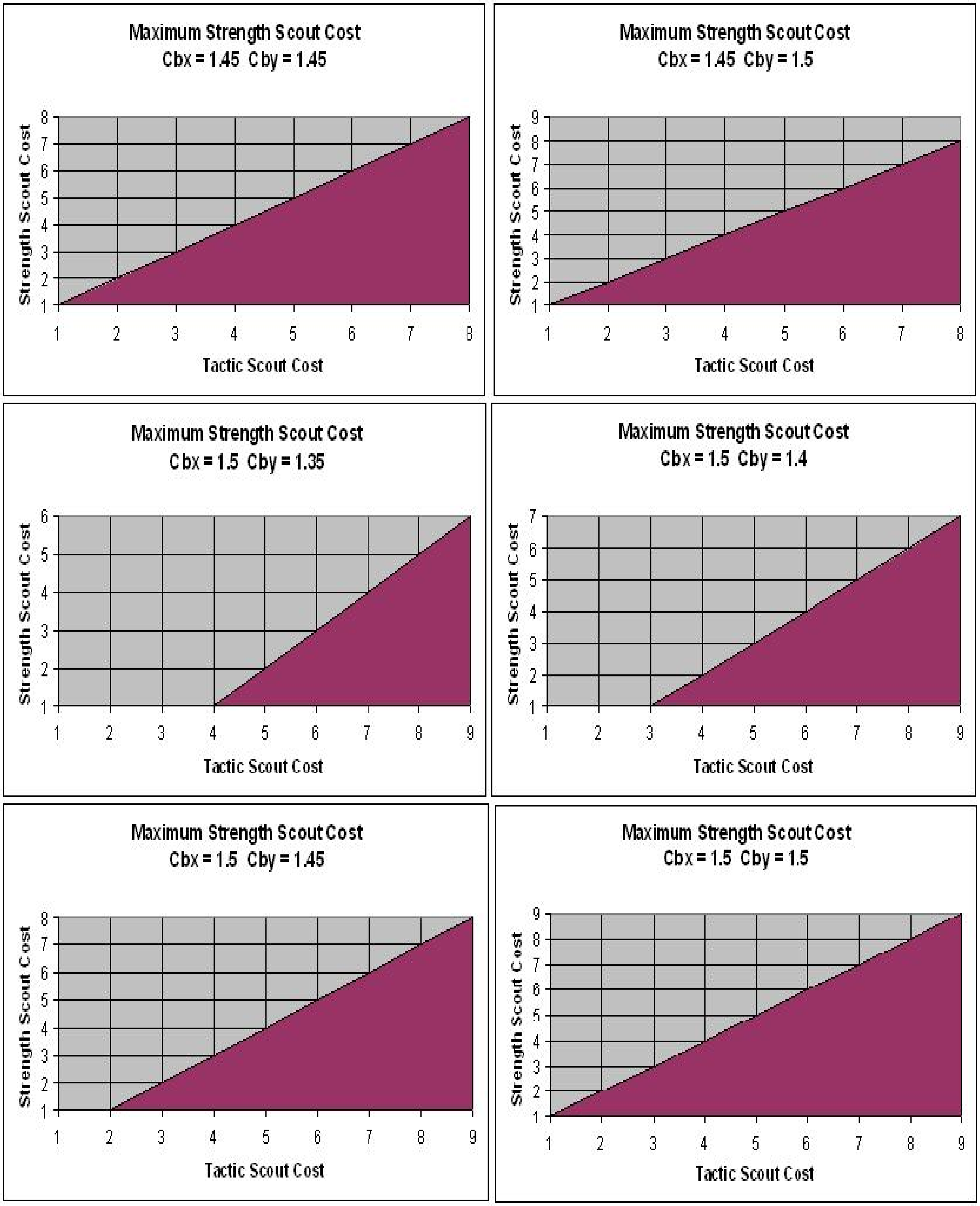
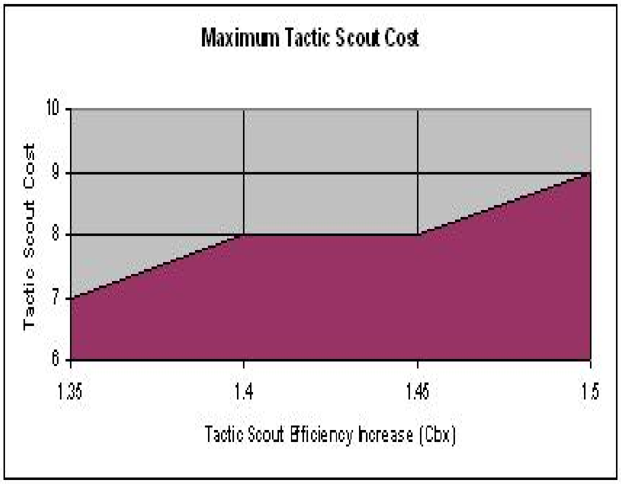
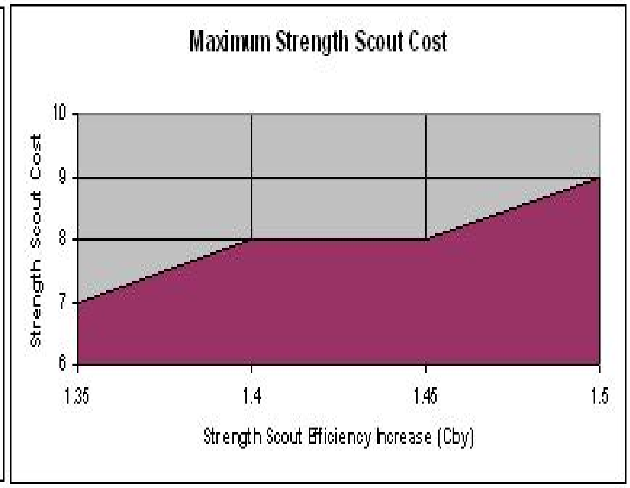


Scenario 5 Feasibility Region Graphs





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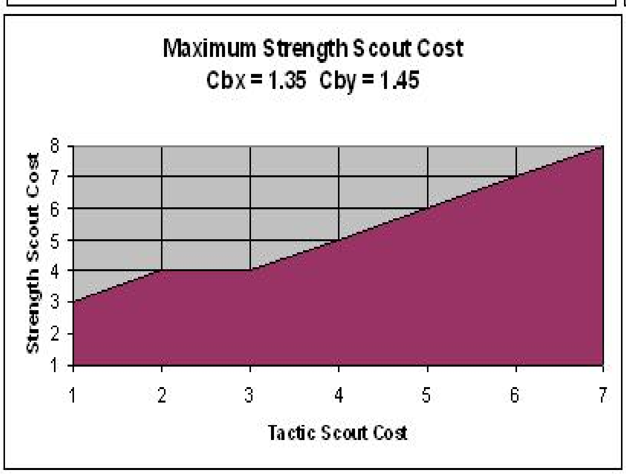

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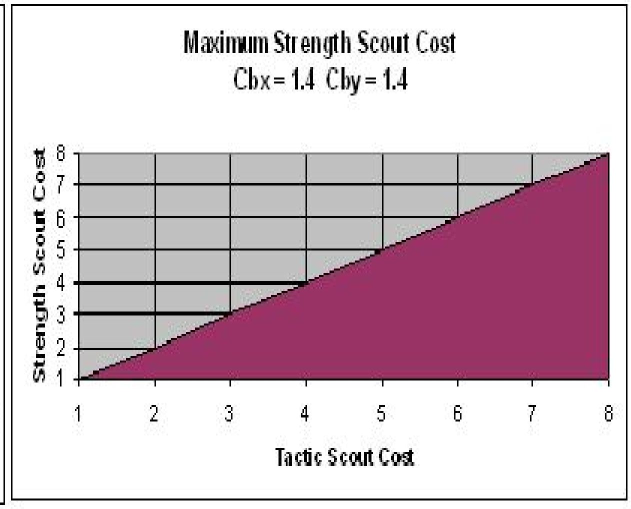

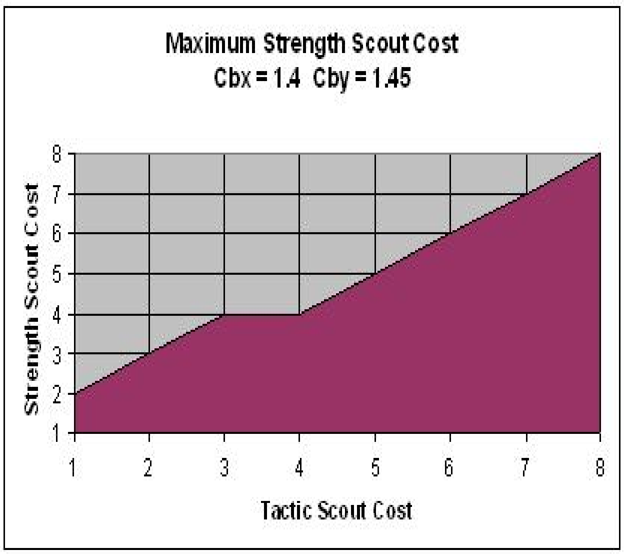

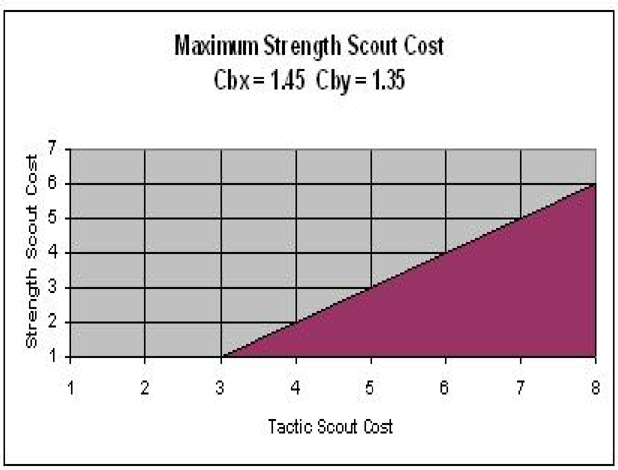

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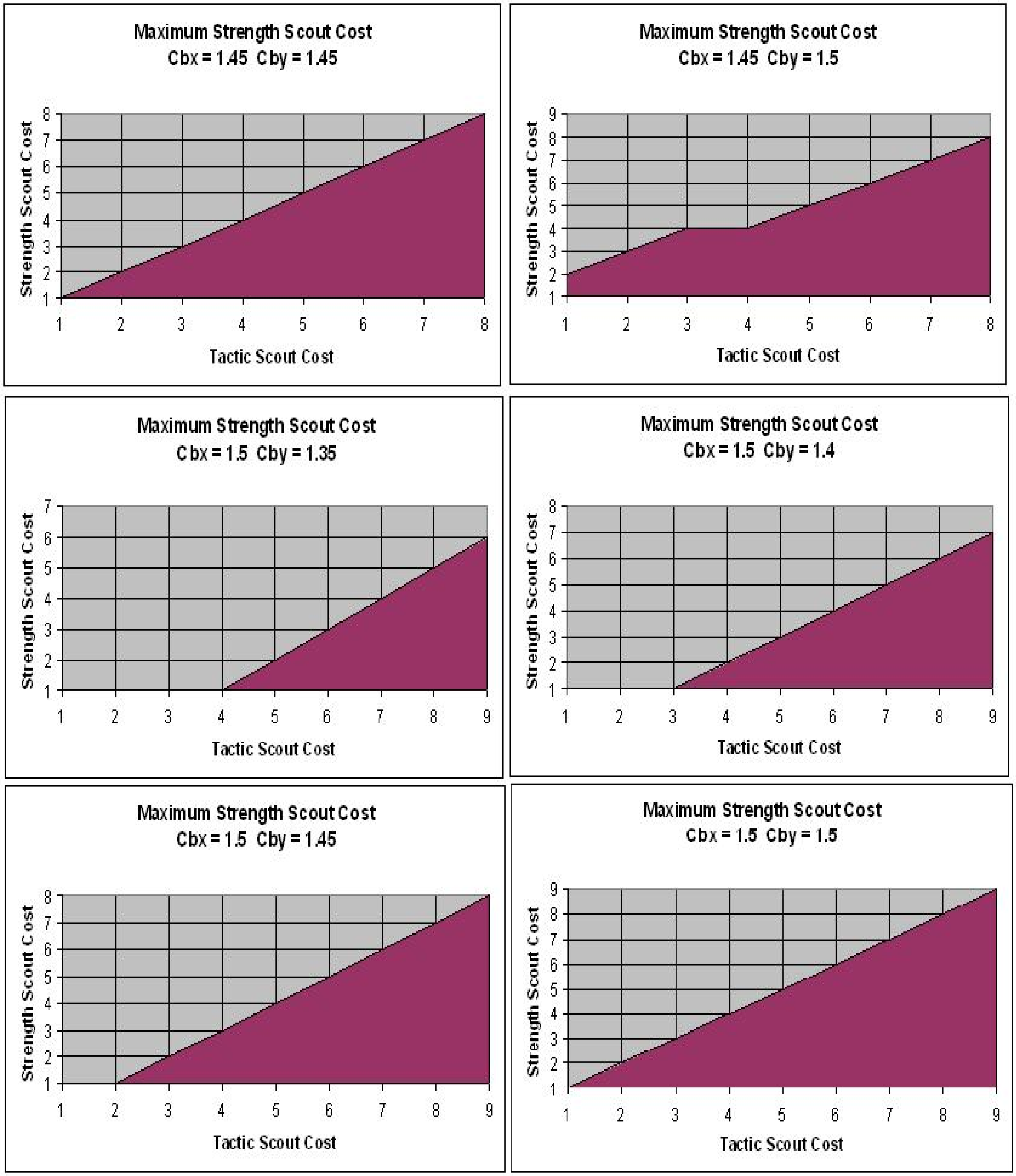

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[1] Darilek, R. E., Perry, W. L., Bracken, J., Gordon J., IV, & Nichiporuk, B. (2001), *Measures of Effectiveness for the Information-Age Army*. Santa Monica, CA: RAND Corporation. Available from World Wide Web: (http://www.rand.org/pubs/monograph_reports/MR1155)

[2] Davis, P. (1995), *Aggregation, Disaggregation, and the 3:1 Rules in Ground Combat*. Santa Monica, CA: RAND Corporation. Available from World Wide Web: (http://www.rand.org/pubs/monograph_reports/MR638)