Visualizing Lagrange multipliers using Geogebra

MAA Session on Demos and Strategies that Enhance Teaching and Learning Mathematics, JMM 2008

Elisha Peterson

United States Military Academy

January 7, 2008

The Lagrange Multiplier Technique

Problem. A farmer standing in a field near his house sees his house catch on fire. There is a lake nearby. He has a bucket with him. What is the shortest path he can take to pick up water from the lake and put out the fire? [as told by Fred Rickey]

Lagrange Multipliers

The *Lagrange multipliers* technique says that, given a function $f(x, y, z)$, the extreme values of f on the set of points $g(x, y, z) = C$ occur (if they exist and g is well-behaved) when

$$\nabla f = \lambda \cdot \nabla g,$$

for some λ, where ∇f is the *gradient* of f.

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Where the idea comes from

The Lagrange multiplier technique depends on the following two facts:

- maxima and minima of functions occur when surfaces are tangent to each other;
- if surfaces are tangent, their normal vectors are parallel;

Both of these facts can be "discovered" by students.

Example. Mount Everest is "tangent" to the set of points 29,029 ft above sea level. An explorer who reaches the top is "perpendicular" to the peak and also parallel to the force of gravity.

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Student perception of Lagrange multipliers

What do students think about the technique?

- The procedure is long and difficult to memorize. [skill]
- It's hard to come up with constraint and objective equations. [skill]
- It's difficult to grasp intuitively why the formula $\mathbf{v} = \lambda \mathbf{w}$ has anything to do with maximization or minimization. [motivation]
- It's okay to just memorize the process without understanding why it works. [intuition]

The **skill** *difficulties are important, but this talk focuses on the* **intuition** *and* **motivational** *aspects.*

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Teaching at West Point

Teaching environment at West Point:

- Cadets typically see this topic at the very end of their freshman year in a course on differential calculus.
- In my course, cadets had the option to use this or the standard "set derivatives equal to zero" technique.
- Cadets are possibly the busiest undergraduates in the country. They tend to be focused on *requirements and procedures* rather than *ideas and concepts*.

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Geogebra

Geogebra
is a free, open source, Java-based dynamic mathematics software, available at http://www.geogebra.org/.

- Intended for use with geometry, algebra, and calculus, but also has been used for graph theory, complex analysis, physics, electronics, and more.
- Large library of materials maintained at http://www.geogebra.org/en/wiki/.
- No native 3D support, but it can be implemented.

Demonstration

Demonstration:

http://www.dean.usma.edu/departments/math/people/Peterson/geogebra/farmer1.html