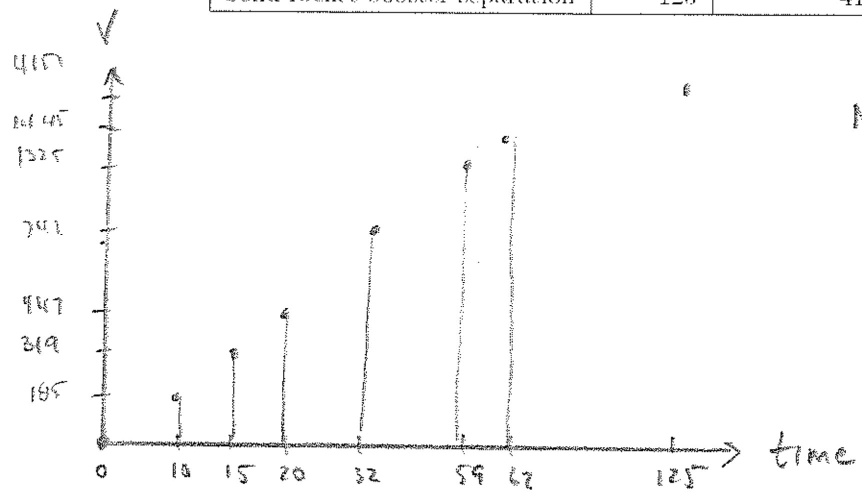
Block I Goal Problems

By the end of this block of instruction, you should be very comfortable analyzing problems similar to these:

1. When we estimate distances from velocity data it is sometimes necessary to use times $t_0, t_1, t_2, t_3, \ldots$ that are not equally spaced. We can still estimate distances using the time periods $\Delta t_i = t_i - t_{i-1}$. For example, on May 7, 1992, the space shuttle Endeavor was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use the data to estimate the height above Earth's surface of the space shuttle Endeavor, 62 seconds after liftoff.

Event	Time(s)	Velocity (ft/s)
Launch	0	0
Begin Roll Maneuver	10	185
End roll Maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

DISCRETE DATA
ΔX IS NOT SAME FOR ALL INTERVALS.



NOT TO SCALE

$n = 6$

LEP

$$\sum_{i=1}^{n} f(x_{i-1})\,\Delta x$$

REP

$$\sum_{i=1}^{n} f(x_i)\,\Delta x$$

$(10-0)\,0 + (15-10)\,185 + (20-15)\,319 + (32-20)\,447 + (59-32)\,742 + (62-59)\,1325$

$= 31{,}893$ ft

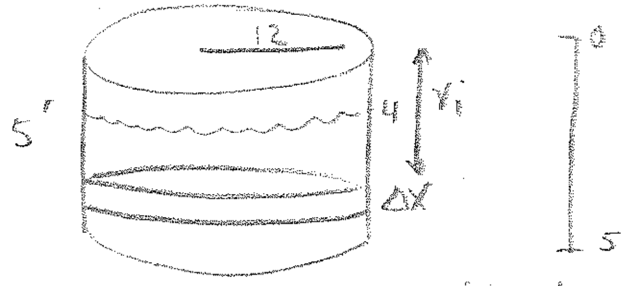
$(10-0)\,185 + (15-10)\,319 + (20-15)\,447 + (32-20)\,742 + (59-32)\,1325 + (62-59)\,1445$

$= 54{,}694$ ft

Trap $= 43{,}293.5$ ft

2. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side? (Use the fact that water weighs $62.5\frac{lb}{ft^3}$)

Assume only have to find Work to top of tank.



$W \approx \sum_{i=1}^{n} W_i$

$\lim_{n\to\infty} \sum_{i=1}^{n} W_i = \int_1^5 (62.5)12^2 \pi x\, d$

x_i - distance i^{th} slice travels

WORK =

Slice = $W_i = F_i d_i$

$W_i = \left(62.5 \frac{lb}{ft^3} (12^2 \cdot \pi \cdot \Delta x) \cancel{ft^3}\right) x_i$ ft

$W_i = 62.5 (12)^2 \pi\, x_i\, \Delta x$

3. The marginal cost function $C'(x)$ is defined to be the rate of change of the production cost function. If the marginal cost of manufacturing x units of a product is $C'(x) = 0.0006x^2 - 1.5x + 8$ (measured in dollars per unit) and the fixed start-up cost is $C(0) = \$1,500,000$, find the cost of producing the first 2000 units and then determine the particular cost function that describes the cost of producing any amount of items, $C(x)$, for this company, based on its start up cost.

GIVEN = RATE OF CHANGE

a) $C(x) = \int C'(x)\, dx$

$= \int .0006x^2 - 1.5x + 8\, dx \Rightarrow \int_0^{2000} .0006x^2 - 1.5x + 8\, dx$

$= 1,384,000$

$+ 1,500,000$

$\$\ 2,884,000$

b) $C(x) = .0002x^3 - .75x^2 + 8x + C$

$c(0) = -1.5$ mill $= c$

$c(x) = .0002(x^3) - .75(x^2) + 8(x) - 1,500,000$

$c(x) = .0002(x)^3 - .75(x)^2 + 8(x) - 1.5$ mill

4. A hot, wet summer is causing a mosquito population explosion in a lake resort area. The number of mosquitos is increasing at an estimated rate of $2200 + 10e^{0.8t}$ per week (where t is measured in weeks). By how much does the mosquito population increase between the fifth and ninth weeks of summer? Determine a function that models the total mosquito population at any week. How many mosquitoes will there be at the end of the summer?

$u = .8t$

$du = .8\,dt$

$\frac{du}{.8} = dt$

a) $\int \text{Rate}\,dt \Rightarrow \int_{t=5}^{t=9} (2200 + 10e^{.8t})\,dt$

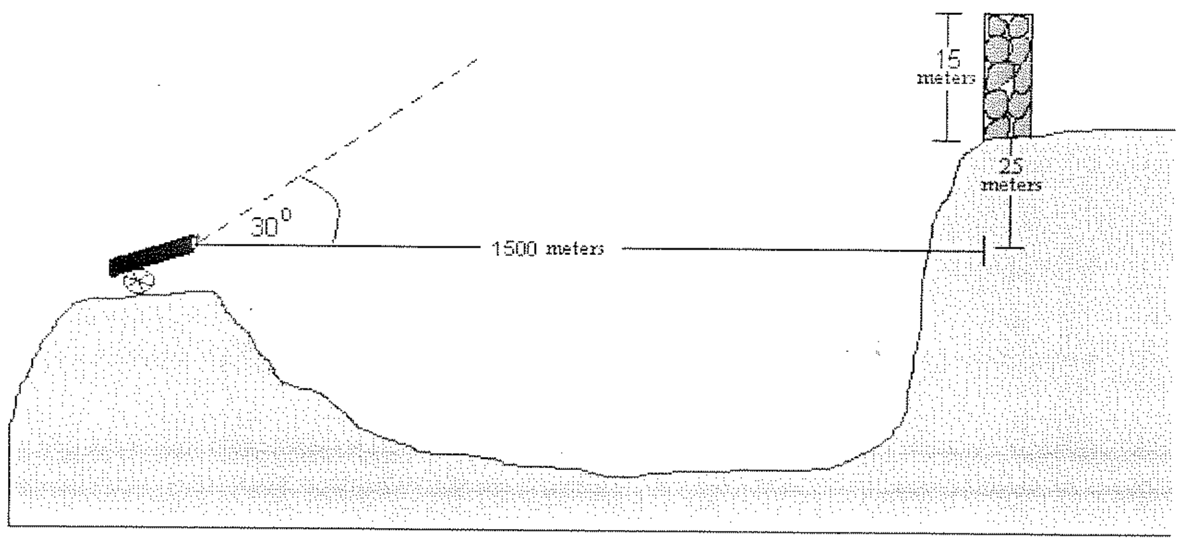
$\int_5^9 2200\,dt + \int_5^9 10e^{.8t}\,dt$

$2200(4) + 12.5e^{.8t} \Big|_{4=u}^{7.2=u}$

$8800 + 12.5(e^{7.2} - e^{4}) \Rightarrow 24860.4$

b) $P(\text{End Summer}) = \int_0^t 2200 + 10e^{.8t}\,dt$

5. A 19th century artillery battery has established its firing position on some high ground 1500m away from an enemy position which is located on a hill top 25m higher in elevation than the battery's location. Additionally, a 15m high wall protects the enemy unit. For the following problems assume a constant acceleration due to gravity of $9.81 \frac{m}{sec^2}$.



(a) Determine a vector function in terms of the initial velocity v_o which represents the location of a cannon ball shot from the battery location at time t. Be sure to clearly identify the origin of the coordinate system you are using.

$r_0 = \langle 0, 0 \rangle$

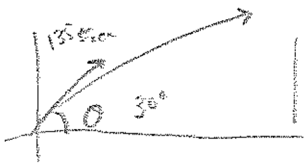
$V_0 = \langle V_0 \cos(\theta), V_0 \sin(\theta) \rangle$

$\vec{a} \langle 0, -g \rangle$

$\vec{v} \langle V_0 \cos\theta, V_0 \sin\theta - gt \rangle$

$\vec{r} \langle V_0 \cos\theta\, t, V_0 \sin\theta\, t - gt^2 \rangle$

(b) Assuming the guns of the battery have a muzzle speed of $135\frac{m}{sec}$ and the guns are prepared to fire at an initial angle of 30°, use your vector function to determine whether or not the cannon balls will clear the wall.



Use $t = 12.83$ to find height.

$\vec{r}(t) = \langle V_0 \cos\theta\, t,\ V_0 \sin\theta\, t - \frac{1}{2} g t^2 \rangle$

We know how to find t, set $x = 1500$m

$V_0 \cos\theta\, t = 1500$

$t = \dfrac{1500}{135\cos(30)} = 12.83$ sec

$V_0 \sin\theta\, t - \frac{1}{2} g t^2 = y$

$135 \sin(30)(12.83) - \frac{1}{2}(9.8)(12.8)^2 =$

$y = 59.4'$

$59.4 > 40$

clears wall!

(c) Again assume that the guns of the battery have a muzzle speed of $135\frac{m}{sec}$ and the guns are prepared to fire at an initial angle of 35°. Assuming the cannon balls clear the wall, what will the speed of the cannon balls be the instant they impact the ground?

speed $= |v(t)| = \sqrt{(V_0 \cos\theta)^2 + (V_0 \sin\theta - gt)^2}$

$= \sqrt{(135\cos 35)^2 + (135 \sin 35 - 9.8t)^2}$

$\text{Speed}(t) = \sqrt{12{,}229.2 + (77.44 - 9.8t)^2}$

We know t @ impact? yes, what will the final position be $\Rightarrow (x, y = 25)$

$y = V_0 \sin\theta\, t - \frac{1}{2} g t^2$

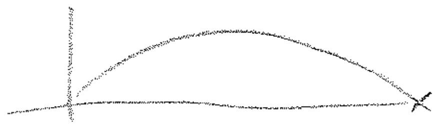
$y = 25 = 135 \sin(35) t - \frac{1}{2}(9.8) t^2$

$t = \cancel{\ }$ or 15.47 sec, use in Speed(15.47)

$\text{Speed}(15.47) = \underline{133.175 \text{ m/s}}$

(d) What is the maximum range of the cannon? (given the same muzzle speed as in part c)

Given same muzzle speed, θ, max range $y = 0$



$(x, y=0)$

Do not consider scenario where $y = 25$ at impact.

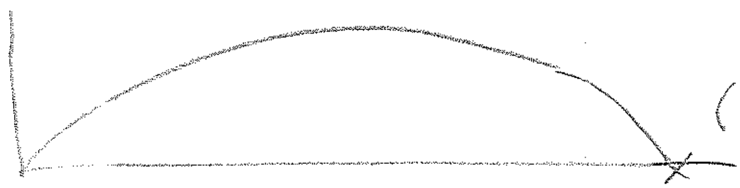
$\cancel{y}^{0} = \cancel{y_0}^{0} + V_0 \sin\theta\, t - \frac{1}{2} g t^2$

$135 \sin(35) t - \frac{1}{2}(9.8) t^2 = 0$

$t = 15.8\,\text{sec}$

Solve $x \Rightarrow$ $x = V_0 \cos\theta\, t = 135(\cos(35))(15.8) = \underline{1747.5\,m}$

(e) If the projectile does impact at the cannon's maximum range with a parabolic trajectory, what is the total distance the cannon ball traveled through the air?



$(1747.5m, 0m)$

Arclength $\Rightarrow$ distance of curve

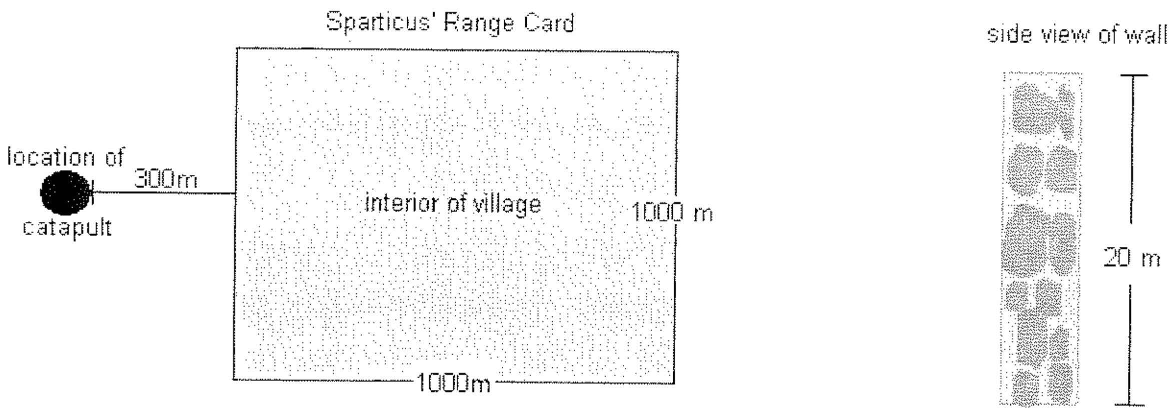
Distance = Speed * Time

Speed is variable $\Rightarrow$ $\text{Speed}(t) = \sqrt{(V_0 \cos\theta)^2 + (V_0 \sin\theta - gt)^2}$

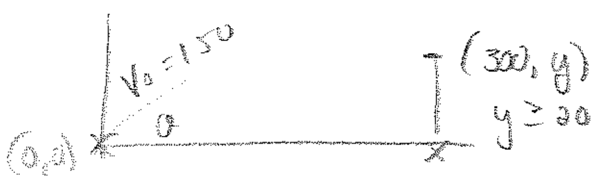
$$\int_{0\,\text{sec}}^{15.8\,\text{sec}} \sqrt{(135\cos(35))^2 + (135\sin(35) - 9.8t)^2}\, dt$$

Length of flight path = $\underline{1880.98\,m}$ ANS

6. The year is 27 AD, you and your buddy, Sparticus, are placing a village, in Gaul, under siege. A stone wall that is $20m$ high surrounds the village. You can get no closer than $300m$ to the wall. Sparticus notices that all of the buildings in the village are made of wood and thinks it would be a good idea to catapult heated rocks over the wall into the village to start it burning and end the siege. You have a catapult that generates an initial velocity of $150\frac{m}{s}$. Sparticus wants you to tell the men at what angle to set the catapult so that the rocks land in the village. Note that the village is perfectly square with each of the four walls being $1000m$ long. Find the range of angles that you can give to the men to insure success. Concern yourself only with minimum and maximum ranges. Assume you are centered on one wall and your rock will travel in a direction perpendicular to the wall.(See Figure Below)



Situation I



Want fireball to clear wall

$x = 300 = V_0 \cos\theta\, t$

$t = \frac{300}{V_0 \cos\theta} = \frac{300}{150\cos\theta}$

$y = 20 = V_0 \sin\theta\, t - \frac{1}{2} g t^2$

Solve Mathematica

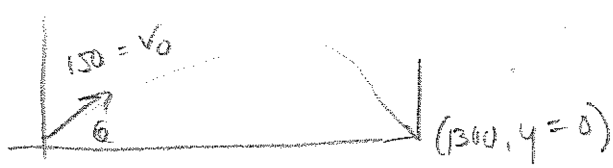
Solve $[\{x(t), y(t)\}, \{t, \theta\}]$

$2.0 \leq t \leq 30.4$

$.13 \cdot \frac{180}{\pi} \leq \theta \leq 1.5 \cdot \frac{180}{\pi}$

$7.4 \leq \theta \leq 85.98$

Situation II



Want fireball to land short of wall.

$.3 \cdot \frac{180}{\pi} \leq \theta \leq 1.2 \frac{180}{\pi}$

$17.9 \leq \theta \leq 68.8$

$7.9 \leq \theta \leq 17.9$

$68.8 \leq \theta \leq 85.98$

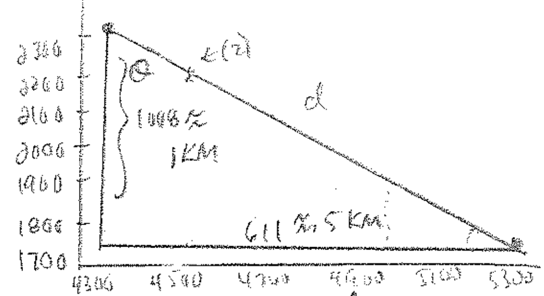
7. A tank moves from grid location WL43542335 to grid location WL53621724 over flat terrain. All distances are measured in meters. The average speed of the tank is $5\frac{km}{hr}$.

$5\,km \cdot \frac{1000\,m}{1\,Km} \cdot \frac{1\,hr}{60\,min} = 83.3\,\frac{m}{min}$

   (a) Graph the path of the tank. Indicate direction of motion.
   (b) Develop a set of parametric equations that describes the tank's movement.
   (c) What are the coordinates of the tank 2 minutes after it starts moving?
   (d) If the tank's horizontal (east/west) location is 5000, what is it's vertical location?
   (e) Assuming the tank continues to move in the same direction and at the same speed, what will the coordinates of the tank be 1 hour later? (assume the location is on the same map sheet)

a) $x = [5362 - 4354] = 1008$

$y = [1724 - 2335] = -611$



b) PARAMETRIC EQUATIONS

$x = x_0 + (V_{0x})t$ $\qquad$ $y = y_0 - (V_{0y})t$

$x(t) = 4354 + 83.3\cos[\theta]t$ $\qquad$ $y(t) = 1724 - 83.3\sin(\theta)t$

$\tan\theta = \frac{611}{1008}$

$\theta = 16.7$

c) $x(2) = 4354 + 83.3\cos(16.7)t$ $\qquad$ $y(2) = 2335 - 83.3\sin(16.7)t$

$x(2) = 4513.64$ $\qquad$ $y(2) = 2287.36$

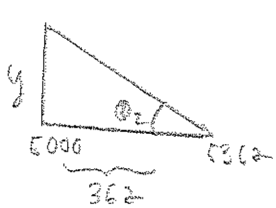
d) $d = \sqrt{(1008)^2 + (611)^2}$

$d = 1178.2\,m$

e) $x(60) = 9143.3$

$y(60) = 905.811$

$90 - \theta_2 = 73.3^\circ$



$\tan(73.3) = \frac{362}{d}$

$d = \underline{108.64}$

So $1724 + 108.64 = \underline{1832.64}$