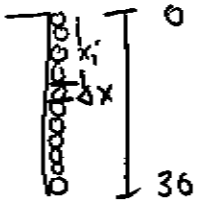
MA205 - Integral Calculus
Lesson 14: Applications I - Work
Board Problems

1. A uniform cable hangs over the edge of a cliff. The cable is $30m$ long and its mass is $45kg$. How much work is required to pull the entire cable to the top of the cliff?

Assumption - coil the cable on top

x_i = distance ith slice travels



$F_i = \rho \cdot \Delta x$

$\rho = \frac{45kg}{30m} = 1.5\frac{kg}{m}$

$F_i = 1.5\frac{kg}{m} \cdot 9.8\frac{m}{s^2} \cdot \Delta x\, m$

$F_i = (1.5)(9.8)\Delta x \ N$

$d_i = x_i$

$W_i = F_i \cdot d_i$

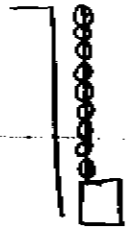
$W \approx \sum W_i$

$\sum (1.5)(9.8)\Delta x\, x_i$

$\lim_{n\to\infty} \sum_{i=1}^{n} W_i =$

$= \int_0^{30} (1.5)(9.8)x\,dx$

2. A chain that weighs 2.5 pounds per foot is used to lift 600 pounds of coal up a mine shaft that is 250 feet deep. Find the work done.



Assumptions - coil chain at top

x_i - distance ith slice traveled

Constant Force $\Rightarrow$ Constant Work

$F = 600 lbs$

$d = 250 ft$

$W = 250(600)\ ft \cdot lb$

Variable Force $\Rightarrow$ Variable Work

$W_{TOTAL} = W_{VAR} + W_{CONSTANT}$

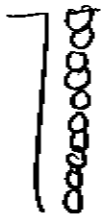
$F = P\Delta x = 2.5\frac{lb}{ft}\Delta x\, ft$

$d = x_i$

$W_i = 2.5 x_i \Delta x \Rightarrow \sum W_i = W$

$\int_0^{250} 2.5x\,dx$

3. Return to the same setting as Problem 1: A uniform cable hangs over the edge of a cliff. The cable is $30m$ long and its mass is $45kg$. This time, calculate how much work is required to pull *one third* of the cable to the top of the cliff.



Assumption: coil cable at top

x_i = distance ith slice travels

$\rho = \frac{45kg}{30m} = 1.5\frac{kg}{m}$

Variable Dist

$F_i = 1.5\frac{kg}{m} \cdot 9.8\frac{m}{s^2} \cdot \Delta x\, m$

$F_i = 1.5(9.8)\Delta x\, N$

$d_i = x_i$

$W_i = 1.5(9.9)x_i\,\Delta x$

$\int_0^{16} 1.5(9.8)x\,dx$

Constant Force Distance

$W = F \times D$

$= \left(1.5\frac{kg}{m} \cdot 9.8\frac{m}{s^2} \cdot 20\right)(10\,m)$

$W = (1.5)(9.8)(20)(10)\ N \cdot m$

$W_{TOTAL} = W_{VAR} + W_{CONSTANT}$