MA205 - Integral Calculus
Lesson 15: Applications II - More Work

1. To calculate the work done by a non-constant force, we must integrate (see equation 4, page 461). However, sometimes we are not given a force and it's not obvious what we need to integrate. In this case we probably need to:

   (a) Draw a picture.

   (b) Make an x-axis in the direction the item will be moving to (or from). You may want to mark a couple of key values along that axis.

   (c) Pick a sample "slice" of liquid in your tank, and suppose it has thickness Δx. Concentrate on moving just that slice of liquid.

   (d) Calculate the work done to move the arbitrary slice. There should be some variables and a Δx in your formula.

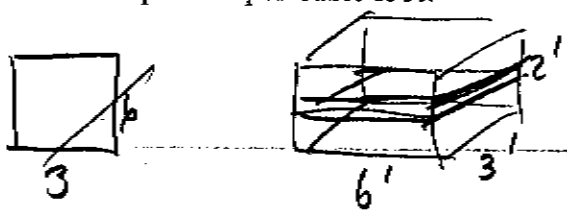
   (e) Think of adding up all the little works in a summation. Give the summation a make-over and turn it into an integral.

   (f) What are the possible x-values you could have chosen for your slice? (i.e. where are your slices "allowed" to come from?) These will give you the limits of integration.

   (g) Now integrate to find the total work.

Problems

1. Suppose we have a rectangular aquarium whose base is 3 feet by 6 feet, and whose height is 2 feet. It is full of water. Your parents are not home. Your little brother whips the wet-dry vac out of the shed and goes to town on the fish tank. Before you can stop the madness, half of the water is pumped out of the tank. How much work has the vacuum done? You may need the fact that water weighs about 62.5 pounds per cubic foot.



Start w/ a slice

$d_i = x_i$ - distance ith slice travels.

$\rho = 62.5 \frac{lbs}{ft^3}$

$F_i = 62.5 \frac{lbs}{ft^3} (6)(3)\Delta x$

$= 62.5(18)\Delta x$ lbs

$W_i = 62.5(18)x\,\Delta x$

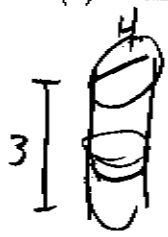
$\Downarrow$

$\int_0^{2'} 62.5(18)x\,dx$

562.5 ft·lbs

2. Same scenario, but what if instead the tank was shaped like a cylinder that has a 4 meter diameter and is 3 meters tall? In SI units, water weighs 9800 Newtons per cubic meter.

(a) If the tank is full of water, how much work will it take to pump all the water out?



$\rho = 9800 \frac{N}{m^3}$

x_i - distance ith slice travels

$F_i = \rho \Delta x = 9800 \frac{N}{m^3} \pi (2)^2 \Delta x$

$d_i = x_i$

$W_i = \overbrace{9800(4)(\pi)}^{F} \overbrace{x}^{d} \overbrace{\Delta x}^{F}$

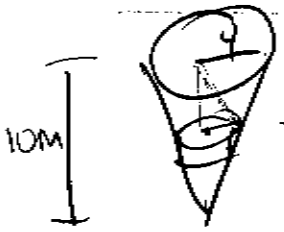
$\int_0^3 9800(4)(\pi) x \, dx = 176400\pi$ N·m

(b) If the tank is only half full of water, how much work will it take to pump all the water out?

$\int_{1.5}^{3} 9800(4)(\pi) x \, dx = 415633$ N·m

3. ~~Do Problem Solving Problem 2 in your student guide.~~

4. Suppose a tank has the shape of an inverted circular cone. (Picture a party hat upside down.) The cone has a height of $10m$ and a base of radius $4m$ and is full of water. Remembering that water weighs $9800\,N/m^3$, find the work required to pump all the water out of the tank.



$\rho = 9800 \frac{N}{m^3}$ $\quad V_i = \pi r^2 \Delta x$

$F_i = \rho V_i = 9800 \frac{N}{m^3} \cdot \pi r^2 \Delta x \, m^3$

Similar Δ's $\quad \frac{r_i}{10 - x_i} = \frac{4}{10}$

$r_i = \frac{2}{5}(10 - x)$

$d_i = x_i$

$W_i = 9800(\pi)\left(\frac{2}{5}(10-x)\right)^2 x \, \Delta x$