MA205

Lesson 19 Vector Functions

1. Suppose a spaceship is traveling through space with an acceleration given by $3\sin(t)\,\mathbf{i} + t\,\mathbf{j} + 3\cos(t)\,\mathbf{k}$.
   a. Find the function that describes the position of the ship, if its initial velocity is $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ and its position after 2 minutes is represented by $\mathbf{i}+8\mathbf{j}+\mathbf{k}$.

GIVEN: $a(t) = \langle 3\sin(t), t, 3\cos(t)\rangle$

$v(0) = \langle 3, 1, -1\rangle$

$r(2) = \langle 1, 8, 1\rangle$

Find: $r(t)$

$v(t) = \int a(t)\,dt = \S$

$v(t) = \langle -3\cos(t)+c, \frac{t^2}{2}+c, 3\sin(t)+c\rangle$

using $v(0)$ solve for c. $\langle 3, 1, -1\rangle$

$v(t) = \langle -3\cos(t)+6, \frac{t^2}{2}+1, 3\sin(t)-1\rangle$

$r(t) = \int v(t)\,dt$

$r(t) = \langle -3\sin(t)+6t+c,$
$\frac{t^3}{6}+t+c,$
$-3\cos(t)-t+c\rangle$

Using $r(2)$ solve for c

$r(t) = \langle -3\sin(t)+6t-8.27,$
$\frac{t^3}{6}+t+4.7,$
$-3\cos(t)-t+1.75\rangle$ ANS

   b. What is the speed of the spaceship at 3 minutes?

Speed is magnitude of velocity

$v(t) = \langle -3\cos(t)+6, \frac{t^2}{2}+1, 3\sin(t)-1\rangle$

$\sqrt{(-3\cos(t)+6)^2 + (\frac{t^2}{6}+1)^2 + (3\sin(t)-1)^2} = \text{Speed}(t)$

$13.45 = \text{Speed}(3)$

2. Suppose a paper airplane which starts at the origin travels through the air with a velocity given by $< \sin(t), t^2, \sqrt{t} >$. When does the airplane achieve a height of 8 feet in the air? (Please, no demonstrations!)

$v(t) = \langle \sin(t), t^2, \sqrt{t}\rangle$

Find time when position function – z component is at 8'

$r(t) = \int v(t)\,dt = \langle -\cos(t)+c, \frac{t^3}{3}+c, \frac{2t^{3/2}}{3}+c\rangle$

Assume starts at origin $\langle 0, 0, 0\rangle$

$r(t) = \langle -\cos(t), \frac{t^3}{3}, \frac{2t^{2/3}}{3}\rangle$

$\frac{2t^{2/3}}{3} = 8$

$t = 41.57$ sec

3. A batter hits the ball with an acceleration vector $a(t) = 18j - 30k$ and an initial velocity of <-90, -30, 4>. The origin of the coordinate system is at field level directly underneath the center of the pitcher's mound, with the positive x axis located from that point toward home plate. If the initial position of the ball is x=50 feet, and y=1 foot, and z=2 feet at t=1 seconds, ~~how far does the ball travel in the next 5 seconds~~. what is $r(5)$?

Given $a(t)$. $v(0)$

Find $pos(5)$

$v(t) = \int a(t)\,dt = \langle 0+c,\ 18t+c,\ -30t+c \rangle$

Using $v(0) = \langle -90, -30, 4 \rangle$

$v(t) = \langle -90,\ 18t-30,\ -30t+4 \rangle$

$r(t) = \int v(t)\,dt$

$r(t) = \langle -90t+c,\ \frac{18}{2}t^2 - 30t + c,\ -\frac{30}{2}t^2 + 4t + c \rangle$

Using $r(1) = \langle 50, 1, 2 \rangle$

$r(t) = \langle -90t+140,\ 9t^2-30t+22,\ -15t^2+4t+11 \rangle$

4. What is the speed of the ball at t=5?

Solve for $r(5)$

$r(5) = \langle -310,\ 97,\ -344 \rangle$

Not realistic

Speed $= |v(t)|$

$= \sqrt{(-90)^2 + (18t-30)^2 + (-30t+4)^2}$

$= \sqrt{(-90)^2 + (18(5)-30)^2 + (-30(5)+4)^2}$