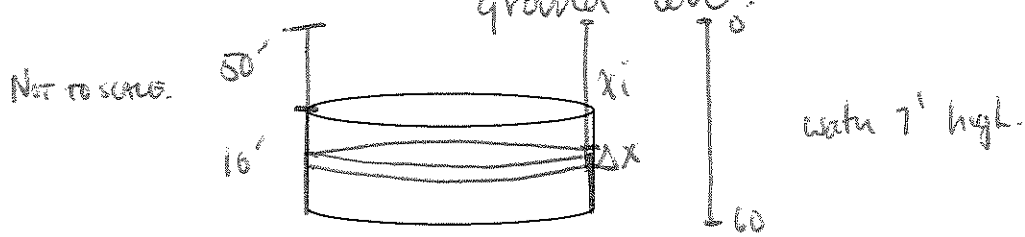
MA205 Graded Homework #2 (30 points)

1. A circular water tank buried **50 feet below ground** has a diameter of 50 ft, the sides are 10 feet high, and the depth of the water is 7 feet. How much works is required to pump all of the water out **to ground level**? Water weighs 62.5 lbs/ft^3.

- Assumptions: pump water to ground $\Rightarrow$ 50 + x_i distance to top
- Define Variables: x_i = distance ith slice travels to ~~top~~ ground level.
- Draw Picture:



- Calculate Work for "*i*-th slice"

$U_i = F_i d_i$ $\qquad F_i = 62.5 \frac{lbs}{ft^3} \cdot (\pi (25)^2 \Delta x) ft^3$

$W_i = (39062.5\pi)\Delta x \, x_i$ $\qquad d_i = x_i$

- Approximate the total Work.

$$W \approx \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} (39062.5\pi) x \, \Delta x$$

- Calculate the exact Work and solve.

$$\lim_{n\to\infty} \sum_{i=1}^{n} W_i = \int_{53}^{60} (39062.5\pi) x \, dx$$

a = min distance ith slice travel 53'

b = max distance " " 60

$$= (\cancel{1.85}\ \cancel{1.54} \times 10^7 \pi) \text{ lbs} \cdot \text{ft}$$

1.54

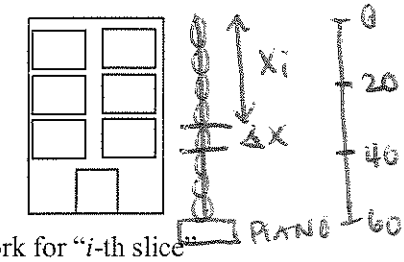
ANS

Also accept $\int_{43}^{50} (39062.5)\pi \, x \, dx$

2. A heavy rope, 60 m long, weights 2.2 kg/m, and hangs over the edge of a building. At the end of the rope is a 2000kg piano that must be moved to the top of the building, how much work is to move the piano to the 3rd floor (2/3 of the way) to the top of the building?

3.

- Assumptions: Assume rope coils at top (no work)
- Define Variables: x_i - distance ith slice travels to top of building
- Draw Picture:



- Calculate Work for "*i*-th slice"

WORK VARIABLE ⇒ ROPE (0 – 40m)
WORK CONSTANT ⇒ ROPE (40-60) + PIANO (40')

$W_i = F_i d_i$ $\qquad F_i = 2.2 \frac{kg}{m} \cdot \frac{9.8 m}{s^2} \cdot \Delta x$

$W_i = 21.56 x_i \Delta x$ $\qquad d_i = x_i$

- Approximate the total Work.

$$W \approx \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} 21.56 x_i \Delta x$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} = \int_0^{40} 21.56x \, dx = 17248 \text{ N}$$

- Calculate the exact Work and solve (how does the integral relate to the approximation?)

WORK VAR

17248 N m

WORK CONSTANT

20m cable

$= (20)(2.2 \frac{kg}{m})(9.8 \frac{m}{s^2})(40) = 17248$ N

Piano

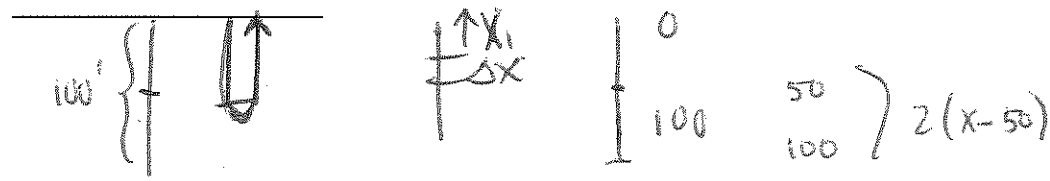
$= (40m)(2000 kg)(9.8 \frac{m}{s^2})$

$= 784000$ N·m

$W = 818496$ N·m

3 4. A 100-ft chain weighs 250 lbs and hangs from the ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that its level with the upper end.

- Assumptions: No slack, work to get to next position
- Define Variables:
  $x_i \Rightarrow$ i^{th} SLICE TRAVELS
- Draw Picture:



- Calculate Work for "*i*-th slice"

$W_i = F_i d_i \qquad F_i = \frac{250(lb)}{100\,ft} \cdot \Delta x\, ft$

$W_i = 2.5\Delta x\,(2)(x_i - 50) \qquad d_i = 2(x_i - 50)$

- Calculate the total Work (approximate)

$$\sum_{i=1}^{n} 2.5(2)(x_i - 50)\,\Delta x = \sum_{i=1}^{n} W_i$$

- Calculate the exact Work and solve (how does the integral relate to the approximation?)

$$\lim_{n\to\infty} \sum_{i=1}^{n} W_i = \int_{50}^{100} 2.5(2)(x-50)\,dx = \underline{\underline{6250}}\ lb \cdot ft$$

also accept $\int_0^{50} 25\,(2x)\,dx$

4. A potato has been fired at an angle and its position function (2-D) follows the parabola $y = -.00032x^2 + x$. Find the length of the path if x and y are measured in feet along the interval $x(10, 20)$? Attach Mathematica code.

$f'(x) = -.00064x + 1$

$$\int_{10}^{20} \sqrt{1 + (f'(x))^2}\, dx$$

$= 14.12$ feet

5. Another potato has been fired straight into the air." Using Newtonian physics and the fact that its time of flight is exactly 6.25 seconds, you determine that the potato has a velocity function $v(t) = -32t + 100\ ft/s$. What is the length of the potato's flight path? Attach Mathematica code.

→ POSITION FUNCTION

$$\int_{0}^{6.25} \sqrt{1 + v(t)}\, dt$$

$= 312.68$ ft

6. A Pitcher throws a curve ball with an acceleration vector $a(t) = 24j - 32k$ and an initial velocity of $120i - 3j + 4k$. The acceleration in the j direction is caused by spin that will result in a curveball. The origin of the coordinate systems is at a field level directly underneath the center of the pitcher's mound, with the positive x axis located from that point toward home plate. If the initial position of the ball is five feet above the origin and one foot to the right, find the y and z coordinates of the ball whenever x=60 feet (ball reaches home plate). Using the fact that home plate is 17 inches wide and one of your team members is the batter, should the pitch be called a strike? State all assumptions.

$v(0) = \langle 120, -3, 4 \rangle$

$r(0) = \langle \cancel{0}\ 0, -1, 5 \rangle$

$\vec{a}(t) = \langle 0, 24, -32 \rangle$

$\vec{v}(t) = \langle c, 24t + c, -32t + c \rangle$

using initial conditions $\vec{v}(0)$

$\vec{v}(t) = \langle 120, 24t - 3, -32t + 4 \rangle$

using initial conditions $\vec{r}(0) = \langle 60, -1, 5 \rangle$

$\vec{r}(t) = \langle 120t + c, 12t^2 - 3t + c, -\frac{32}{2}t^2 + 4t + c \rangle$

$\vec{r}(t) = \langle 120t + \cancel{60}\ 0, 12t^2 - 3t - 1, -16t^2 + 4t + 5 \rangle$

Find when $x = 60$. Then Solve for y, z components

$120t = 60$

$t = 1/2$

$y = 12(\frac{1}{2})^2 - 3(\frac{1}{2}) - 1 = \frac{1}{2} \approx 6$ in

$z = -16(\frac{1}{2})^2 + 4(\frac{1}{2}) + 5$

Assume Strike Zone

z $(2 \text{ to } 4)'$

y $(-8.5 \text{ to } 8.5)''$

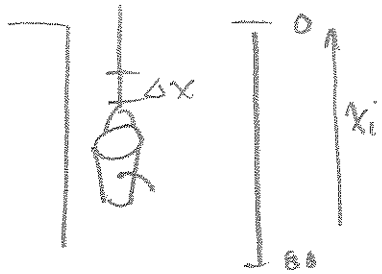
STRIKE!

$(x = 60, y = .5, z = 3)$

1. Bonus1 (3 points): a bucket weights 4lb and a rope of negligible weight are used to draw water from a well that is 80 feet deep. The bucket is filled with 40lb of water and is pulled up at a rate of 2ft/s, but water leaks out of a hole in the bucket at a rate of .2lb/s. Find the work done in pulling the bucket to the top of the well.

GIVEN: Constant F (bucket) + Var F (Water)

Assumptions: x_i distance i^{th} slice travels



$$W_i = F_i d_i \qquad \begin{cases} F_i = (4 + (40 - .1x_i))\Delta x \\ d_i = x_i \end{cases}$$

$$W_i = 4 + (40 - .1x)x\,\Delta x$$

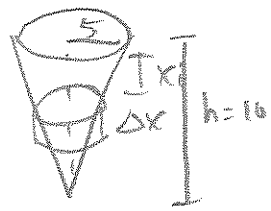
$$W \approx \sum_{i=1}^{n} W_i$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} W_i = \int_0^{80} 4 + (40 - .1x)x\,dx$$

Start: 40 lb H_2O

Rate: $\frac{2'}{s}$ leaks at $.2\frac{lb}{s}$ $\Rightarrow$ $-.1\frac{lb}{ft}$: MATHEMATICA $\Rightarrow$ $\underline{\underline{111253 \text{ lb}\cdot\text{ft}}}$ ANS

2. Bonus2 (3 points): a coffee thermos, which can be modeled by a cone with radius 5ft and height 10 ft is completely filled with Dunkin Donuts coffee. Recall D&D Coffee weighs approximately 34.5 lbs/ft. How much work is required to pump all of the coffee out of the thermos? Hint: see page 533.

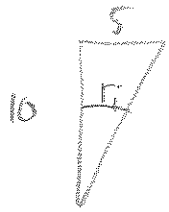


Assumptions: pump to top and done

x_i = distance i^{th} slice travel

$$W_i = F_i d_i \qquad \begin{cases} F_i = 34.5\frac{lb}{ft} \cdot \pi\left(\frac{1}{2}(10-x_i)\right)\Delta x \\ d_i = x_i \end{cases}$$

$$W_i = 34.5\left(\tfrac{1}{2}(10-x_i)\right)\pi x\,\Delta x$$



$$W \approx \sum_{i=1}^{n} W_i$$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} W_i = \int_0^{10} 34.5\left(\tfrac{1}{2}(10-x_i)\right)\pi x\,dx$$

$$\frac{5}{10} = \frac{r_i}{10-x}$$

$$5(10-x) = 10r_i$$

$$\frac{1}{2}(10-x_i) = r_i$$