BC 1 Limits 1 Name: **Solutions**

Nice Limits

Some limits seem obvious. For example, $\lim_{x\to 2}\left(x^2+3x-1\right)=9$, as we would all guess. This is because the function $f(x) = x^2 + 3x - 1$ is a well-behaved function near $x = 2$. In other words, as x gets closer to 2, $f(x)$ gets closer to 9, so we say the limit equals 9. The behavior of the function is predictable.

As long as the function is nice and well-defined in an open interval containing $x = a$, we may evaluate the limit by simply evaluating the function at $x = a$.

(1) $\lim_{x\to 2}(5x-1)$

$\boxed{5(2)-1=\mathbf{9}}$

(2) $\lim_{x\to 3}|x-1|$

$\boxed{|3-1|=\mathbf{2}}$

(3) $\lim_{x\to \pi/6}\dfrac{\cos x}{\sin(3x)}$

$$\boxed{\frac{\cos(\pi/6)}{\sin(3\pi/6)}=\frac{\sqrt{3}/2}{1}=\frac{\sqrt{3}}{2}}$$

Not-so-Nice Limits $\boxed{\dfrac{0}{0}}$

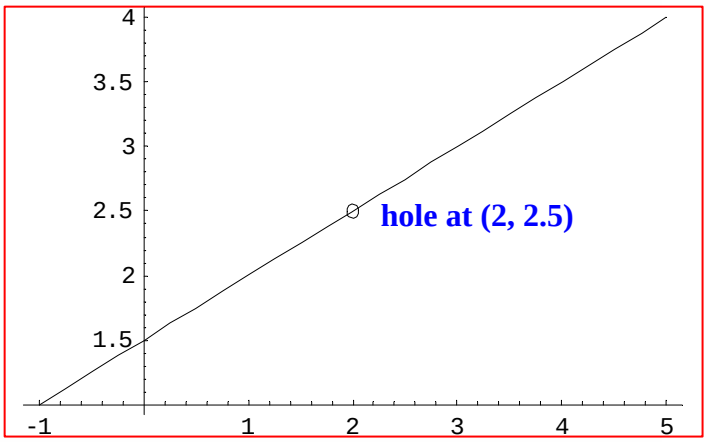
Unfortunately, lots of limits aren't quite so obvious and lots of functions are not well-behaved at all points. Specifically, the limit we evaluate to find any derivative yields an expression undefined when $h = 0$. In addition, we may wish to study the behavior of a function <u>near</u> a value of x where the denominator is 0. In short, we need to understand more about limits.

Ex: $\lim_{x\to 2}\dfrac{x^2+x-6}{2x-4}$. The function here is $f(x)=\dfrac{x^2+x-6}{2x-4}$ and the point in question is $x = 2$. Look at this problem numerically first. Use the table ~~(with Ask)~~ on your calculator to fill in the following chart.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	**2.45**	**2.495**	**2.4995**	undef	**2.5005**	**2.505**	**2.55**

What does this suggest? $\boxed{\lim_{x\to 2} f(x)=2}$

Now sketch the graph of f.
(Try zoom-decimal. Use xres = 1 on the TI-89.)



http://staff.imsa.edu/math/sharing/BCworksheets/Limits1.pdf

Continued: Now, it's time to do some algebra. Simplify the expression and complete the following:

$$\lim_{x\to 2}\frac{x^2+x-6}{2x-4} = \boxed{\lim_{x\to 2}\frac{(x-2)(x+3)}{2(x-2)} = \lim_{x\to 2}\frac{(x+3)}{2} = \frac{5}{2}}$$

Does your analytic work confirm your graph and numerical work?

Many of these limits may be calculated by doing some algebra first, simplifying, and then evaluating the resulting expression. Do the following analytically. Confirm your work by using tables and/or graphics.

(4) $\lim_{x\to -1}\frac{x^2-3x-4}{x^2+7x+6}$ $\boxed{= -1}$

(5) $\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}$ $\boxed{= 6x}$

(6) $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ $\boxed{= \lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{x}+2)} = \lim_{x\to 4}\frac{1}{(\sqrt{x}+2)} = \frac{1}{4}}$

(7) $\lim_{x\to 5}\frac{\frac{2}{x}-\frac{2}{5}}{x-5}$ $\boxed{= \frac{-2}{25}}$

The following limits cannot be determined by simple algebraic methods. Use graphs and tables to estimate/guess each limit.

(8) $\lim_{x\to 0}\frac{\sin(3x)}{2x}$ $\boxed{\approx 1.5}$

(9) $\lim_{x\to 1}\frac{\ln x}{x-1}$ $\boxed{\approx 1}$

(10) $\lim_{x\to 2}\frac{3^x-9}{x-2}$ $\boxed{\approx 9.9}$