Name: K E Y

Section: ____________

MA205 Lesson 30 Graded Homework Rectangular Regions (L28&30)

(20points)

Instructions: Be sure to answer each part of the question, label each part of the question appropriately.

1. **In your own words, who is Fubini and what did he do (a) (do not simply write down the theorem from the text)? Will you be able to use Fubini's Theorem every time you calculate a double integral(b)? Why or why not?**

(a) Italian mathematician who developed a theorem that says if a function f is continuous over a rectangular region (Domain), then the limits of integration do not change when you change the order of integration. Meaning that if given the following integral:

$$\int_c^d \int_a^b f(x,y)dxdy$$

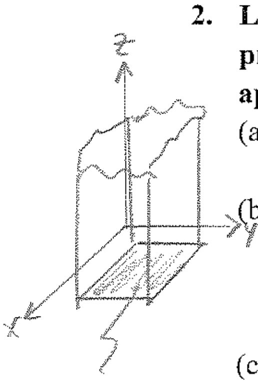
Then if we reversed the order of integration, then values of a,b,c,d would not change. So

$$\int_1^3 \int_2^5 f(x,y)dxdy = \int_2^5 \int_1^3 f(x,y)dydx$$

This only happens when the limits of integration in both the x and y direction are defined by constant functions (ex: y=3).

(b) No, this theorem is limited to rectangular regions and we will look at more complicated non-rectangular domains. This does not mean that if the domain is not a rectangular region then we can't switch the order of integration, it means that we simply cannot switch the order and not change the limits of integration as the values of a,b,c,d will change because they will be defined by functions of a variable.

2. **Lesson 28 and 30 deal with Rectangular Regions, explain what a rectangular region is (a), provide a graphical example (b), and explain how it is used in calculating both the approximation and exact values for volume (c)?**



DOMAIN RECTANGULAR

(a) When we say rectangular region, we are talking about the shape of the Domain being rectangular.

(b) Approximation-we break the rectangular region (domain) into cubes of length and width, Δx & Δy, and height $f(x,y)$ to approximate the volume under the surface. The approximation estimates the volume under the surface as it relates to the x and y axis. Because the domain is rectangular, we are able to use cubes to approximate the volume.

(c) Exact-the rectangular region (domain) establishes the limits of integration in both the x and y direction.

3. Solve the following by hand. What do you notice about the results? Explain why this happens.

(1) $\int_1^4 \boxed{\int_0^1 (xe^{x^2} + \frac{1}{y}) dx} dy$ (2) $\int_0^1 \int_1^4 (xe^{x^2} + \frac{1}{y}) dy dx$

$$\int_{x=0}^{x=1} xe^{x^2} + \frac{1}{y} \, dx$$

$$\int_{x=0}^{x=1} xe^{x^2} dx + \int_{x=0}^{x=1} \frac{1}{y} dx$$

$u = x^2$

$du = 2x \, dx$

$dx = \frac{du}{2x}$

$$\int_{x=0}^{x=1} x \frac{e^{u}}{2\cancel{x}} du \qquad \int_{x=0}^{x=1} \frac{1}{y} dx$$

$$\frac{1}{2} e^{x^2} \Big|_0^1 \qquad \frac{x}{y} \Big|_0^1$$

$$\frac{1}{2}[e - 1] + [\frac{1}{y} + 0]$$

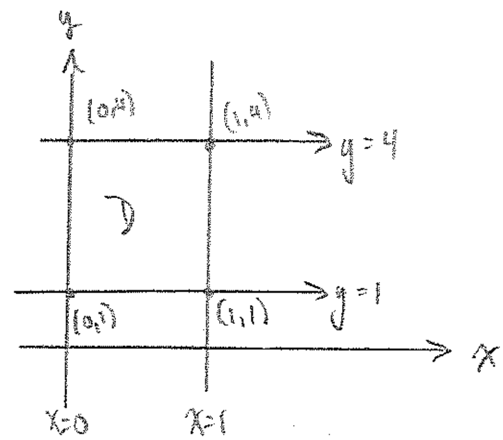
$$\int_{y=1}^{y=4} \left(\frac{1}{2}(e-1) + \frac{1}{y}\right] dy$$

$$\frac{1}{2}(e-1)y + \ln(y) \Big|_1^4$$

$$\left[\frac{1}{2}(e-1)4 + \ln(4)\right] - \left[\frac{1}{2}(e-1)1 + \cancel{\ln(1)}^{0}\right]$$

$\underline{3.9637}$ ANS

RESULTS ARE SAME



RECTANGULAR REGION →

CONTINUOUS ON DOMAIN

if $y = 0$, then what would happen? Would not be continuous.