KEY

**1.** For what nonzero values of $k$ does the function $y = \sin kt$ satisfy the differential equation?

$y'' + 4y = 0$

$y' = k\cos(kt)$

$y'' = -k^2 \sin(kt)$

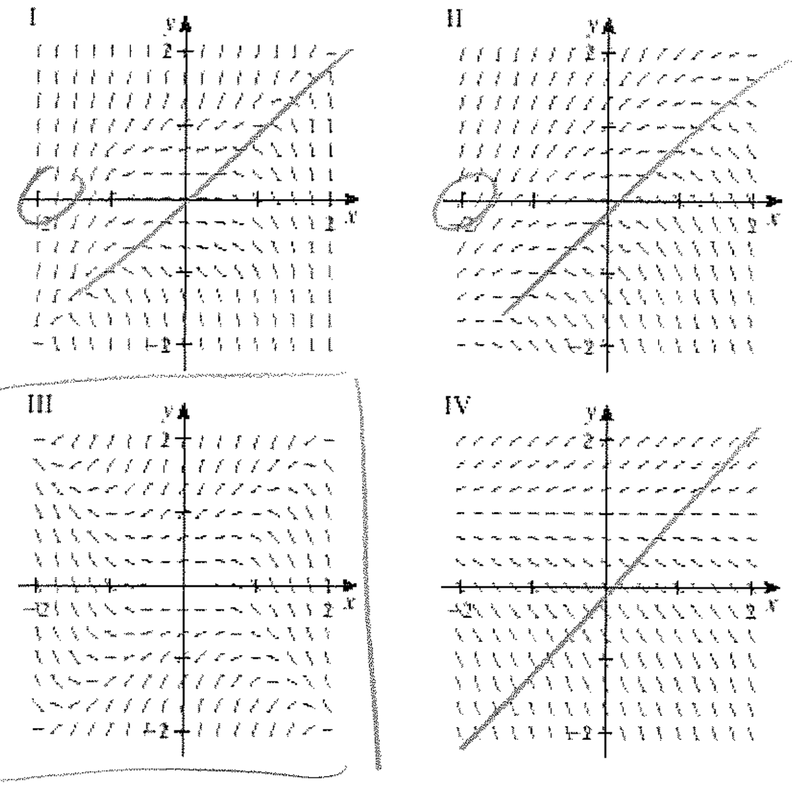
$-k^2 \sin(kt) + 4\sin(kt) = 0$

$\sin(kt)[-k^2 + 4] = 0$

$k = \pm 2, 0$

**2.** Select a direction field for the differential equation $y' = y^2 - x^2$ from a set of direction fields labeled I-IV by hand. Pick at least 5 points for x, y, y' to prove your answer.

| x | y | y' |
|---|---|---|
| 0 | 0 | 0 |
| -2 | 0 | -4 |



**3.** Classify the following differential equation in terms of linearity, order, homogeneity:

$xy' + e^x - x^7 y = 0$

- linear – no products, power, function
- non-homogenous – $e^x$
- $1^{st}$ order – $y'$

**4.** Which equation does the function $y = e^{-3t}$ satisfy? Select the correct answer. Show all work.

a. $y'' + y' + 12y = 0$
b. $y'' + y' - 12y = 0$
c. $y'' - y' - 12y = 0$
d. $y'' - y' + 12y = 0$
e. $y'' - 3y' + 12y = 0$

$y = e^{-3t}$

$y' = -3e^{-3t}$

$y'' = 9e^{-3t}$

$9e^{-3t} - (-3e^{-3t}) - 12e^{-3t} = 0$

+

**5.** a) $y = Ce^{4x^2}$ is the family of solutions to the differential equation $y' = 8xy$. Find the solution that satisfies the initial condition $y(0) = 8$.

$y = Ce^{4x^2} \Rightarrow y(0) = 8$

$8 = Ce^{4(0)^2}$ $\quad C = 8$

Particular Solution

$y = 8e^{4x^2}$ Ans

b) $y = \dfrac{1}{10x + C}$ is the family of solutions of the differential equation $y' = -10y^2$, find the solution that satisfies the initial condition $y(0) = 6$.

$y = \dfrac{1}{10x + C}$ $\quad 6 = \dfrac{1}{C}$ $\quad y = \dfrac{1}{10x + 1/6}$

$6 = \dfrac{1}{10(0) + C}$ $\quad C = \dfrac{1}{6}$ $\quad$ Ans

**6.** One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction of the population who have heard the rumor and the fraction who have not heard the rumor. Let's assume that the constant of proportionality is $k = 0.04$. Write a differential equation that is satisfied by $y$.

$R(t)$ - # people who have heard rumor by time $t$ (Variable)

$$\frac{dR}{dt} = k\left(\frac{R(t)}{P}\right)\left(1 - \frac{R(t)}{P}\right)$$

$P$ - # people in population (constant)

**7.** **Newton's Law of Cooling** states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Suppose that a roast turkey is taken from an oven when its temperature has reached 170 ° F and is placed on a table in a room where the temperature is 70 ° F. If $u(t)$ is the temperature of the turkey after $t$ minutes, then Newton's Law of Cooling implies that $\dfrac{du}{dt} = k(u - 70)$. This could be solved as a separable differential equation. Another method is to use slope fields. If the temperature of the turkey is 165 ° F after half an hour, what is the temperature after 45 min?

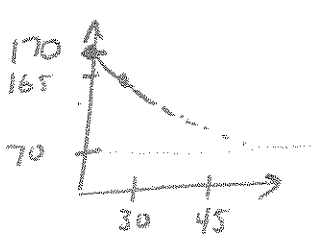
$u(t)$ - Temp of turkey after $t$

$T_s = 70°$

$t = 0,\ 170°$

$t = 30,\ 165$

$t @ 45\ ?$




$\dfrac{du}{dt} = k(u - 70)$

$\dfrac{du}{dt} = \dfrac{-5}{30}$ $\quad \dfrac{170 - 165}{0 - 30}$

$\dfrac{du}{dt} = -\dfrac{1}{6}$

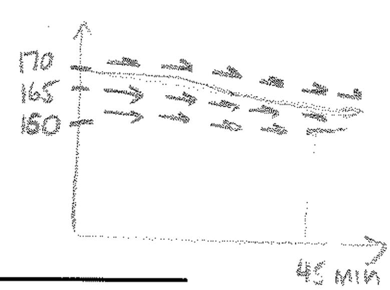
$-\dfrac{1}{6} = k(165 - 70)$

$-\dfrac{1}{6} = k(95)$

$k = -.0017$

DEPLOT

$\dfrac{du}{dt} = -.0017(u - 70)$




$\approx 163°$