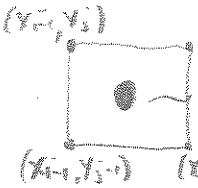
Recall



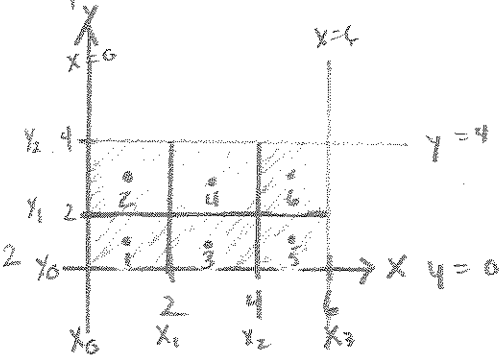
$$\bar{x}_i = \left(\frac{x_{i-1} + x_i}{2}\right)$$

$$\bar{y}_j = \left(\frac{y_{j-1} + y_j}{2}\right)$$

1. Estimate the volume of the solid that lies below the surface $z=xy$ and above the rectangle $R = \{(x,y)\} \mid 0 \le x \le 6, 0 \le y \le 4\}$.

a) Use the Midpoint Rule to estimate the volume of the solid with m=3, n=2.

Step 1: Draw the Domain



(2 rectangles) $n = 2$

$\Delta y = \frac{d-c}{n} = \frac{4-0}{2} = 2$

$m = 3$

(3 Rectangles)

$\Delta x = \frac{b-a}{m} = \frac{6-0}{3} = 2$

cubes $= m \times n = 3 \times 2 = 6$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f(\bar{x}_i, \bar{y}_j)\,\Delta x\,\Delta y \qquad \Delta x \Delta y = (2)(2) = 4$$

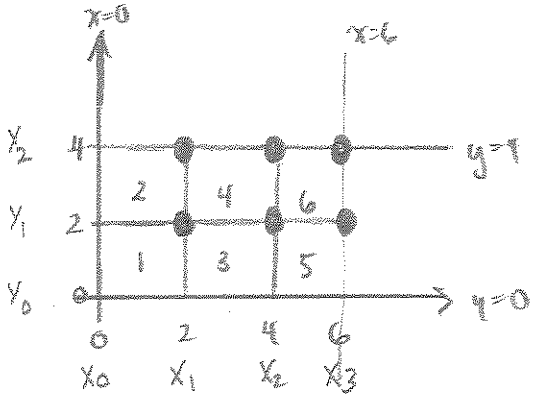
$$\Delta x \Delta y \sum_{i=1}^{m}\sum_{j=1}^{n} f(\bar{x}_i, \bar{y}_j)$$

$$4\left[\underset{①}{f(x_1,y_1)} + \underset{②}{f(x_1,y_2)} + \underset{③}{f(x_2,y_1)} + \underset{④}{f(x_2,y_2)} + \underset{⑤}{f(x_3,y_1)} + \underset{⑥}{f(x_3,y_2)}\right]$$

$$4\left[f(1,1) + f(1,3) + f(3,1) + f(3,3) + f(5,1) + f(5,3)\right]$$

$$4\left[1 + 3 + 3 + 9 + 5 + 15\right] = 144$$

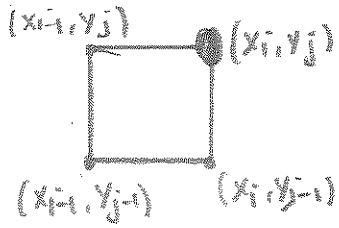
b) Use Riemann sum taking the sample point to the upper right corner of each sub-rectangle.



$\Delta y = 2$

$\Delta x = 2$

Recall Upper Right



ONE RECTANGLE ⇒ ONE CUBE

cubes $= m \times n = 6$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_j)\,\boxed{\Delta x \Delta y} = \text{constant}$$

$2(2) = 4$

$$\Delta x \Delta y \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_j)$$

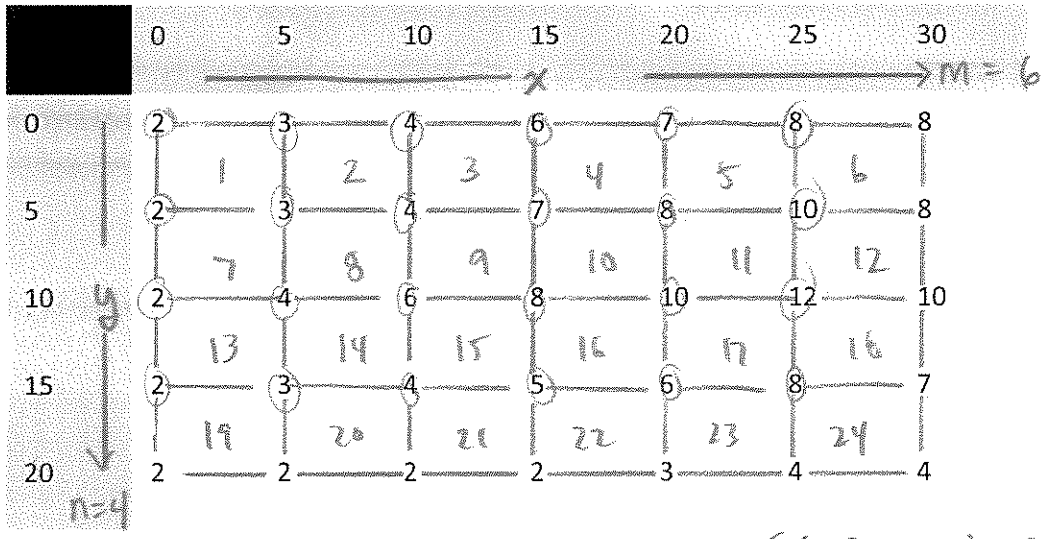
$$\Rightarrow \Delta x \Delta y \left[\underset{①}{f(2,2)} + \underset{②}{f(2,4)} + \underset{③}{f(4,2)} + \underset{④}{f(4,4)} + \underset{⑤}{f(6,2)} + \underset{⑥}{f(6,4)}\right]$$

$$4\left[4 + 8 + 8 + 16 + 12 + 24\right]$$

$$= 248$$

2. A 20ft X 30ft swimming pool is filled with water. The depth is measured at 5-ft intervals, starting at one corner of the pool, and the values are recorded in the table below. Estimate the volume of water in the pool

STEP 1 ⇒ CONNECT THE DOTS ⇒ MAKE RECTANGULARS



EX: Box 1

upper L (2) — (3) upper R

Lower L (2) — (3) Lower R

$\Delta x = \frac{30-0}{6} = 5$

$\Delta y = \frac{20-0}{4} = 5$

$\sum\sum f(x_i, y_j)\Delta x \Delta y$

GO WITH UPPER LEFT!

CUBES = m x n = 4 x 6 = 24

$\Delta x \Delta y = 25$ So

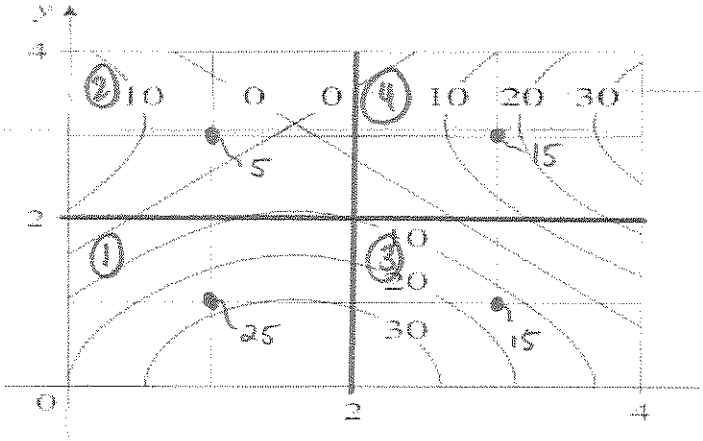
25 (Add up circled values)

25 (134) = 3350

3. A contour map is shown for a function f on the square R =[0,4] X [0,4].

a) Use the Midpoint Rule with m=n=2 to estimate the value of $\iint_R f(x,y)dA$

Note bottom and left columns not used. Why?



$\Delta y = \frac{4-0}{2} = 2$

$\Delta x = \frac{4-0}{2} = 2$

$f(\bar{x}_i, \bar{y}_j)$

① ② ③ ④

$\Delta x \Delta y = 4$ So $4(25 + 5 + 15 + 15) = 240$