What is a differential equation?

Consider the question: How do I translate a physical phenomenon into a set of equations which describes it?

Relate rates (derivatives) of unknown to a function to each other to model problems ⇒ find the function

How can we solve a DE?

1) Graphically
2) Analytically
3) Numerically

Define the following:

1) Order – magnitude of derivative of dependent variable

Independent
dependent variable

2) Linearity – PPF (No products, powers, or functions of dependent variable

Tricks
Multiply out

3) Homogeneity – are all terms in terms of dependent variable or one of its derivatives

For the following problems: Classify in terms of order, linearity, and homogeneity.

1. $y'' - y' - 42y = 0$

order = 2
linear
homogeneous

2. $y' = 4xy$.

order = 1
linear
homogeneous

3. $\frac{dP}{dt} = 1.8P\left(1 - \frac{P}{5{,}100}\right)$ ⇒ $1.8P - \frac{1.8P^2}{5100}$

order – 1
homogeneous
non. linear

4. $\frac{dy}{dt} = y^4 - 13y^3 + 40y^2$. ⇒ order = 1
homogeneous
non linear

5. $y' = y^3 - 4y$ ⇒ order = 1
homogeneous
non-linear

6. $y' = xe^{-\sin x} - y\cos x$

order = 1
non-homogeneous
linear

7. $\frac{dx}{dt} = 1 - t + x - tx$

order = 1
non-homogeneous
linear

8. **Newton's Law of Cooling** $\frac{du}{dt} = k(u - 85)$.

order = 1
non-homogeneous
linear

9. Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.07P - 0.0007P^2$ where t is measured in weeks.

order = 1
homogeneous
non-linear

10. The Pacific halibut fishery has been modeled by the differential equation $\frac{dy}{dt} = ky\left(1 - \frac{y}{K}\right)$ where $y(t)$ is the biomass (the total mass of the members of the population) in kilograms at time t (measured in years), the carrying capacity is estimated to be $K = 30 \times 10^9$ kg and $k = 0.07$ per year.

order = 1
non-linear
homogeneous

11.

Let $\frac{dP(t)}{dt} = 0.1P\left(1 - \frac{P}{890}\right) - \frac{1,870}{89}$.

order = 1
non-linear
non-homogeneous

12. Consider the differential equation $\frac{dP(t)}{dt} = 0.6P\left(1 - \frac{P}{900}\right) - c$ as a model for a fish population, where t is measured in weeks and c is a constant.

order = 1
non-linear
non-homo

13. $y' + y\ln x = x^3 y^3$

order = 1
nonlinear
homogeneous

14. An object with mass m is dropped from rest and we assume that the air resistance is proportional to the speed of the object. If $s(t)$ is the distance dropped after t seconds, then the speed is $v = s'(t)$ and the acceleration is $a = v'(t)$. If g is the acceleration due to gravity, then the downward force on the object is $mg - cv$, where c is a positive constant, and Newton's Second Law gives $m\frac{dv}{dt} = mg - cv$.

order = 1
linear

non-homogeneous