Block II Objectives and Calendar

Block II Objectives

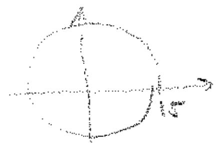
1. Develop a mathematical model given information on how something changes in two dimensions.
   (a) Articulate the assumptions required for mathematical modeling.
   (b) Understand when to make an appropriate assumption.
2. Understand and use $\sum$ and Δ Notation with more than one independent variable.
3. Estimate "volumes".
   (a) Use tabular data to approximate "volumes" numerically.
   (b) Use level curves to approximate "volumes" numerically.
4. Understand the definition of an iterated integral and how to compute them.
   (a) Understand how the accumulation process and the iterated integral are related through the limiting process.
   (b) Calculate an iterated integral over a rectangular region.
   (c) Calculate an iterated integral over a general region.
   (d) Given an iterated integral, reverse the order of integration.
5. Become familiar with polar coordinates.
   (a) Transform an iterated integral in cartesian coordinates to an equivalent iterated integral in polar coordinates.
   (b) Understand the meaning of the polar rectangle and how it leads to $r dr d\theta$.
6. Apply iterated integrals (using either cartesian or polar coordinates) to determine:
   (a) Volume and average volume.
   (b) Center of mass.
   (c) Population and average population.

Block II Goal Problems

By the end of this block of instruction, you should be very comfortable analyzing problems similar to these:

1. A small yard sprinkler distributes water in a circular pattern of radius 15ft. It supplies water to a depth of $e^{-\sqrt{r}}$ feet per hour at a distance of r feet from the sprinkler.
   (a) What is the total amount of water supplied per hour to the region inside the circle of radius R ft centered at the sprinkler?
   (b) Determine an expression for the average amount of water per hour per square foot supplied to the region inside the circle of radius R ft.
   (c) Compute the average amount of water per hour, per square foot, that is supplied to the yard at the sprinklers maximum range.

1 Draw Domain



$f(r,\theta) = e^{-\sqrt{r}} \frac{ft}{hr}$

$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=R} e^{-\sqrt{r}} \, \underline{r} \, dr \, d\theta$$

$\frac{ft}{hr} \cdot ft^2$

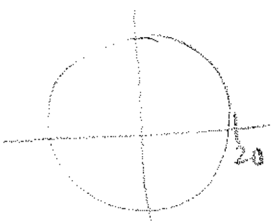
when $R = 15$ $\quad \underline{\underline{40.8}}$ ANS

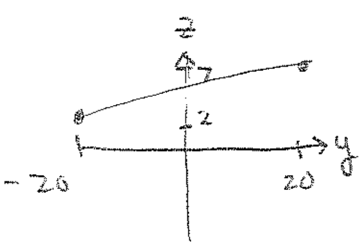
2 Average $= \frac{1}{\text{Area Domain}} \iint r \, dr \, d\theta$

$= \frac{40.8}{\pi(15)^2} \frac{ft^{\cancel{3}}}{hr} \cdot \frac{1}{\cancel{ft^2}}$

$= \underline{\underline{.0577 \frac{ft}{hr}}}$ ANS

2. A swimming pool is circular with a 40-ft diameter. The depth is constant along east-west lines and increases linearly from 2 ft at the south end to 7 ft at the north end. What is the volume of water in the pool?





x- constant

Δ depth $= 7-2 = 5$

$\Delta y = 40-0 = 40$

$\frac{\text{rise}}{\text{run}} = \frac{\Delta z}{\Delta y} = \frac{5}{40} = \frac{1}{8}$

$(z-7) = \frac{1}{8}(y-20)$

$z = \frac{1}{8}y - \frac{5}{2} + \frac{14}{2}$

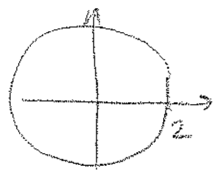
$z = \frac{1}{8}y + \frac{9}{2}$ $\approx$ convert to polar

$\frac{1}{8} r \sin\theta + \frac{9}{2}$

$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=20} \left(\frac{1}{8} r \sin\theta + \frac{9}{2}\right) r \, dr \, d\theta$$

$\underline{\underline{5654.87}}$

3. Electric charge is distributed over a circular disk of radius 2 meters so that the charge density at the point (x, y) is $\sigma(x, y) = x^2 + y^2$ (measured in coulombs per square meter). Find the total charge on the disk.

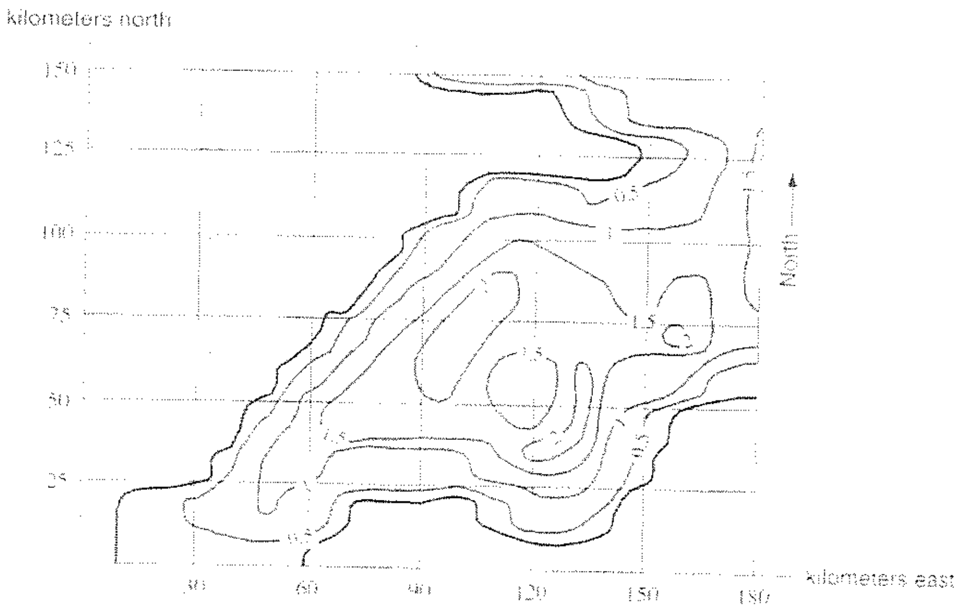


Do in polar

$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} r^2 \, r \, dr \, d\theta$$

$$= 25.1327 \text{ COULOMBS}$$

4. Given the contour plot below of the population density of foxes in a region estimate the total fox population in the region. Form upper and lower estimates for the population. What is the average population density for the region?



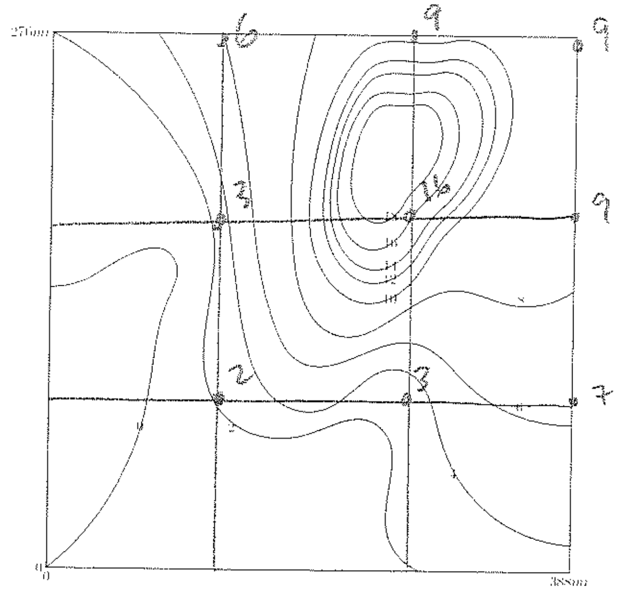
Upper Estimate $= 2(150)(180)$

Lower Estimate $= 0$

Total $(150)(180) = 2700$ FOXES (Average Upper/Lower)

$$\text{Avg}\,\# = \frac{2700}{(150)(180)} = \underline{\underline{1}}$$

5. The contour map below shows the snowfall in inches that fell on the State of Colorado on a particular day. Determine the average amount of snowfall for the state of Colorado, given this map. Estimate the total amount of snowfall for the state.



$m = 3$ $\quad \Delta x = \frac{388}{3} = 127.3$ mi

$n = 3$ $\quad \Delta y = \frac{276}{3} = 92$ mi

$$\Delta x \Delta y \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_i, y_i)$$

$$\Delta x \Delta y \left[2 + 3 + 6 + 3 + 16 + 9 + 7 + 9 + 9 \right]$$

$$\Delta x \Delta y \left[64 \right] = (127.3)(92)(64) = 7\,495\,42.4$$

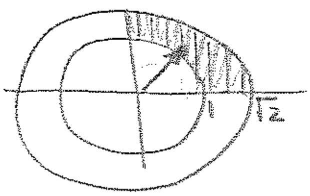
mi^2 · in

$$\text{Average} = \frac{1}{\text{Area}_D \cancel{\text{mi}^2}} \; 7\,495\,42.4 \cancel{\text{mi}^2} \cdot \text{in} = \underline{\underline{7.0 \text{ in}}}$$

$\text{Area}_D = (388)(276)$

$\bar{x} = \frac{1}{m} \iint_D y \,\sigma(x,y)\, dA \qquad \bar{y} = \frac{1}{m} \iint x \,\sigma(x,y)\, dA$

6. A thin disk occupies the region inside the circle $x^2 + y^2 = 2$ but outside the circle $x^2 + y^2 = 1$ in the first quadrant. Find the center of mass if the density at any point is inversely proportional to its distance from the origin.



$\rho(x,y) = \frac{K}{\sqrt{x^2+y^2}}$ or $\frac{K}{\sqrt{r^2}}$ $\frac{K}{r}$

$\bar{x} = \frac{1}{.65K} \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=1}^{r=\sqrt{2}} r\cos\theta \, \frac{K}{\cancel{r}} \cancel{r}\, dr\, d\theta$

$M = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=1}^{r=\sqrt{2}} \frac{K}{\cancel{r}} \cancel{r}\, dr\, d\theta$

$\bar{y} = \frac{1}{.65K} \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=1}^{r=\sqrt{2}} r\sin\theta \, \frac{K}{r}\, r\, dr\, d\theta$

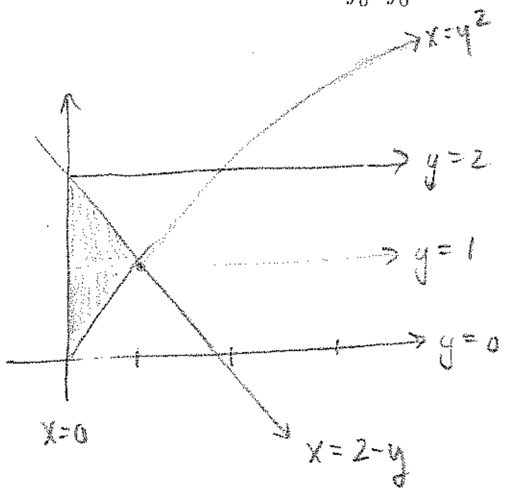
$M = .65K$

$(\bar{x}, \bar{y}) = (.77, .77)$

$r = 1.09 \quad \theta = \pi/4$

7. Determine a single iterated integral that would be equivalent to the sum of iterated integrals also provided below.

$$\int_0^1 \int_0^{y^2} f(x,y)\, dx dy + \int_1^2 \int_0^{2-y} f(x,y)\, dx dy \qquad x = 2-y$$



$$\int_{x=0}^{x=1} \int_{y=\sqrt{x}}^{y=2-x} f(x,y)\, dy\, dx$$

ANS