MA205 Lesson 18

Vector Functions

1. Evaluate the following definite and indefinite integrals by hand.

   (a) $\int_1^2 \left(-\sin(t)\mathrm{i} + 9t^2\mathrm{j} + 3\mathrm{k}\right) dt$

   $+\cos(t)\Big|_1^2 \hat{i} + 3t^3\Big|_1^2 \hat{j} + 3t\Big|_1^2 \hat{k}$

   $\left(\left(\cos(2) - \cos(1)\right)\hat{i} + 21\hat{j} + 3\hat{k}\right)$

   (b) $\int_0^4 \langle 10e^t, -\cos(t), \sqrt{t}\rangle\, dt$

   $10e^t\Big|_0^4 \hat{i} - \sin(t)\Big|_0^4 \hat{j} + \frac{2}{3}t^{\frac{3}{2}}\Big|_0^4 \hat{k}$

   $\left(\left(10e^4 - 10\right)\hat{i} - \left(\sin(4) - \sin(0)\right)\hat{j} + \left(\frac{2}{3}4^{\frac{3}{2}}\right)\hat{k}\right.$

   (c) $\int \left(t^{1/3}\mathrm{i} + \frac{3}{2}t^{2}\mathrm{j} + \frac{3}{t}\mathrm{k}\right) dt$

   $\left(\frac{3}{4}t^{4/3}\hat{i} + \frac{1}{2}t^3\hat{j} + 3\ln|t|\hat{k} + C\right)$

   (d) $\int \langle -t^{-4}, 2.75, 0.5e^t\rangle\, dt$

   $\left(+\frac{1}{3}t^{-3}\hat{i} + 2.75t\hat{j} + .5e^t\hat{k} + C\right)$

2. Suppose $\mathbf{r}'(t) = 2\mathbf{i} + 14t\mathbf{j} + 7\sqrt{t}\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{j} - \mathbf{k}$. Find $\mathbf{r}(t)$.

$r(t) = \int \vec{r}'(t)dt = 2t\hat{i} + 7t\hat{j} + \frac{14}{3}t^{\frac{3}{2}}\hat{k} + C$

$r(0) = 0\hat{i} + (1)j + (-1)K$

So $\hat{i} \Rightarrow \cancel{2t}^{0} + C = 0$

$C = 0$

$\hat{j} \Rightarrow \cancel{7t}^{0} + C = 1$

$C = 1$

$\hat{k} \Rightarrow \cancel{\frac{14}{3}t^{3/2}} = -1 \quad C = -1$

Therefore =

$r(t) = 2t\hat{i} + (7t+1)\hat{j} + \left(\frac{14}{3}t^{\frac{3}{2}} - 1\right)\hat{k}$

Ans

$$\vec{r}(t) = (-2\cos(t)+5)\hat{i} + (-\sin(t))\hat{j} + (3t+\pi)\hat{k}$$

ANS

3. If $\mathbf{r}'(t) = \langle 2\sin(t), -\cos(t), 3\rangle$ and $\mathbf{r}(\pi) = \langle 5, 0, \pi\rangle$, find $\mathbf{r}(t)$.

$$\vec{r}(t) = \int \vec{r}'(t)\,dt = $$

$$\vec{r}(t) = -2\cos(t)\hat{i} - \sin(t)\hat{j} + 3t(\hat{k}) + C$$

$$r(0) = 5\hat{i} + 0\hat{j} + \pi\hat{k}$$

$i \Rightarrow C - 2\cos(0) = 5$ $\quad j = -\sin(t) + C = 0$ $\quad k = 3t^{0} + C = \pi$

$C - 2 = 5$ $\quad -\sin(0) + C = 0$ $\quad C = \pi$

$C = 7$ $\quad C = 0$

4. Suppose the velocity of a particle at any time t is given by $\mathbf{v}(t) = (t^2-1)\mathbf{i} + 10t^3\mathbf{j} - 5t\mathbf{k}$.

(a) Find the vector function that describes the acceleration of the particle at any time t.

(b) If the initial position of the particle is represented by $2\mathbf{i} - \mathbf{j} + 17\mathbf{k}$, find the vector function that describes the position of the particle at any time t.

a) acceleration is $v'(t)$

b) Find the vector function → indefinite integral

a) $$\frac{d(\vec{v}(t))}{dt} = 2t\hat{i} + 30t^2\hat{j} - 5\hat{k} = \vec{a}(t)$$

b) $\vec{r}(0) = 2\hat{i} - j + 17K$ assume $t=0$

Vector Function $\Rightarrow$ $$\int \vec{v}(t)\,dt = \left(\frac{1}{3}t^3 - t\right)\hat{i} + \frac{10}{4}t^4\hat{j} - \frac{5}{2}t^2\hat{k}$$

$$\vec{r}(t) = \left(\frac{1}{3}t^3 - t + c\right)\hat{i} + \left(\frac{10}{4}t^4 + c\right)\hat{j} - \left(\frac{5}{2}t^2 + c\right)\hat{k}$$

$\vec{r}(0) =$ $\quad c = 2$ $\quad C = -1$ $\quad C = 17$

So $$\vec{r}(t) = \left(\left(\frac{1}{3}t^3 - t + 2\right)\hat{i} + \left(\frac{10}{4}t^4 + (-1)\right)\hat{j} - \left(\frac{5}{2}t^2 + 17\right)\hat{k}\right)$$

ANS