Integral Review

5 Anti-Derivatives you must know:

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \quad \textit{Note: Absolute Value signs!!}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x dx = e^x + C$$

Example Problems (where you do not need to use u-substitution):

$\int -\frac{3}{2}\cos(x) + \frac{1}{2}\sin(x)\,dx \qquad -\frac{3}{2}\sin(x) - \frac{1}{2}\cos(x) + C$

$\int -\frac{3}{2x} + \frac{1}{2x^2}\,dx \Rightarrow \int -\frac{3}{2}\frac{1}{x} + \frac{1}{2}x^{-2}\,dx \Rightarrow -\frac{3}{2}\ln|x| + \frac{1}{2}x^{-1} + C$

$\int -\frac{3}{2}e^x - \frac{1}{2}e^x + \frac{1}{4x^5}\,dx \qquad -\frac{3}{2}e^x - \frac{1}{2}e^x - \frac{1}{16}x^{-4} + C$

$\int \frac{4p - 3e^p}{2}\,dp \Rightarrow \int \frac{4p}{2} - \frac{3e^p}{2}\,dp \Rightarrow p^2 - \frac{3}{2}e^p + C$

$\int \frac{4y - 3y^4}{2y}\,dy \Rightarrow \int 2\frac{\cancel{4y}}{\cancel{2y}} - \frac{3y^{4\,3}}{2y}\,dy \Rightarrow 2y - \frac{3}{8}y^3 + C$

$\int -\frac{4\theta - 3\theta^2}{2\theta^4}\,d\theta \qquad \int -\frac{4\theta}{2\theta^4} + \frac{3\theta^2}{2\theta^4}\,d\theta = \int -2\theta^{-3} + \frac{3}{2}\theta^{-2}\,d\theta \Rightarrow \theta^{-2} - \frac{3}{2}\theta^{-1} + C$

U-Substitution ("The un-due of the Chain Rule")

$$U - substitution \ldots\ldots \int f(g(x))g'(x) = \int f(u)du$$

How do I re-write the function "f(x)" in terms of "u" to get a function "f(u)". Make sure all x terms cancel out (or are replaced by u-terms). You should not have any "x" terms remaining once you convert. If you do, you picked the wrong "u" or made an error.

Some common sense rules:

1) Find the function within the function (or is there something in the function that looks different?) If yes, try to set u = to the piece that looks different.
2) If you haven't found the function within the function, then identify the highest exponent (maybe it is hidden and it is really the function with the function). If I take the derivative of it, then will I get the remaining piece of the function (multiplied by a constant)? If yes, try to use it as "u".
3) If you think I found the right u, and still have an x *or some variation of it*, can I use what I defined "u" to be, to get rid of the "x"?

Thought process on integration:

1) Try to break the problem down into its simplest form, go to step 2.
2) Try to use the anti-derivatives you know to integrate, if it doesn't work Step 3.
3) Try U-substitution (see common sense rules), if it doesn't work Step 4.
4) If you can't use u-substitution, use Mathematica (if allowed).

Examples (ranging from easy to hard):

$\int -\frac{3}{2}x\cos(x^2) + \frac{1}{2}x^2\sin(x^3)\,dx$ $\quad u = x^2,\ du = 2x\,dx \quad u = x^3,\ du = 3x^2\,dx \quad \int -\frac{3}{2}\cos(u)\frac{du}{2} + \int \frac{1}{2}\sin(u)\frac{du}{3}$

$-\frac{3}{4}\sin(u) + -\frac{1}{6}\cos(u) + C$ ANS

$\int (x^2 + x + 1)e^{(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x)}dx$ $\quad u = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x,\ du = x^2 + x + 1\,dx \quad \int e^u\,du \Rightarrow e^u + C$ ANS

$\int -\frac{3}{2}xe^{2x^2}dx$ $\quad u = 2x^2,\ du = 4x\,dx \quad \int -\frac{3}{2}e^u\frac{du}{4} \Rightarrow -\frac{3}{8}e^u + c$ ANS

$\int \frac{2y}{4 - 3y^2}dy$ $\quad u = 4 - 3y^2,\ du = -6y\,dy \quad \int \frac{2}{u}\frac{du}{(-6)} \Rightarrow \int -\frac{1}{3u}du \Rightarrow \frac{-\ln|u|}{3} + c$ ANS

$\int \frac{2\theta^3}{4 + 3\theta^4}d\theta$ $\quad u = 4 + 3\theta^4,\ du = 12\theta^3 d\theta \quad \int \frac{2}{12}\cdot\frac{1}{u}\,du \Rightarrow \frac{1}{6}\ln|u| + c$ ANS

$\int -\frac{(4 - 3e^{p^2})p}{2}dp$

$\int -\frac{4}{2}p\,dp + \int \frac{3}{2}pe^{p^2}\,dp \Rightarrow u = p^2,\ du = 2p\,dp$

$\int -p^2 + c + \int \frac{3}{2}\cdot\frac{e^u}{2}\,du$

$-p^2 + \frac{3}{4}e^{p^2} + c$