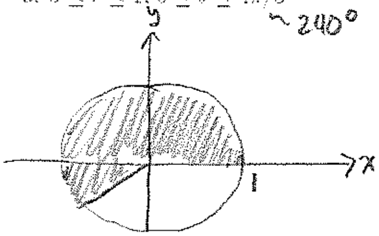
MA205 Lesson 34 Polar Regions

DRAW THE REGION OF INTEGRATION.

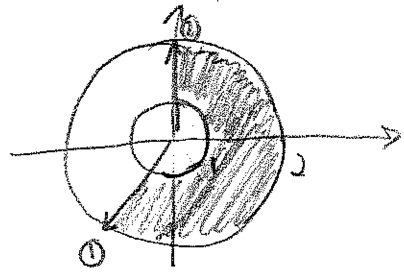
1. An obvious region that is better described by polar coordinates is a *polar rectangle*. Sketch the following polar rectangles.

a. $0 \le r \le 1, 0 \le \theta \le 4\pi/3$ ~ 240°

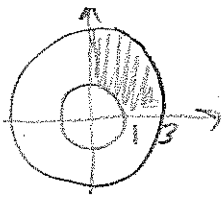


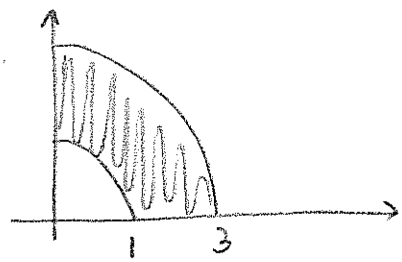
b. $1 \le r \le 2, -3\pi/4 \le \theta \le \pi/2$

$-\frac{3\pi}{4} = \frac{-3(180)}{4} = -135$

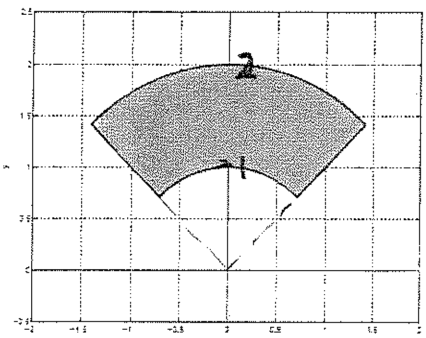


2. Suppose R is the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. Sketch this region, and describe it using inequalities in terms of r and θ.





3. Integrate the function $f(x, y) = 1 - x^2 - y^2$ over the region pictured below using polar coordinates.



$\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$

$1 \le r \le 2$

$$\int_{\theta=\frac{\pi}{4}}^{\theta=\frac{3\pi}{4}} \int_{r=1}^{r=2} 1 - r^2 \boxed{r\,dr\,d\theta}$$

$$\int_{\theta=\frac{\pi}{4}}^{\theta=\frac{3\pi}{4}} \int_{r=1}^{r=2} (r - r^3)\,dr\,d\theta$$

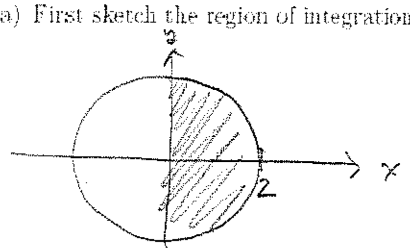
- Remember, when working in polar coordinates the "dA" in the integral $\iint_R f(x,y)\,dA$ is equal to $\underline{r}\,dr\,d\theta$! (or $r\,d\theta\,dr$, depending on the order you choose).

1. Evaluate $\displaystyle\iint_D xy\,dA$ where D is the disk centered at the origin and having radius 3.

$$\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=3} (r\sin\theta)(r\cos\theta)\,r\;dr\,d\theta$$

2. Let's use polar coordinates to evaluate the following integral: $\displaystyle\int_0^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{x^2+y^2}\,dy\,dx$.

   (a) First sketch the region of integration. Mathematica may help.



   (b) Now describe the region in polar coordinates. This example should be a polar rectangle.

$$\int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}}\int_{r=0}^{r=2} e^{r^2}\,r\,dr\,d\theta$$

   (c) Next, re-write the integral in terms of polar coordinates, including the limits of integration. Then integrate!

$u = r^2$

$du = 2r\,dr$

$dr = \dfrac{du}{2r}$

$$\int_{r=0}^{r=2}\frac{e^{u}}{2}\,dr\,d\theta$$

$$\frac{e^{r^2}}{2}\Bigg|_{r=0}^{r=2}$$

$$\int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}}\left[\frac{e^4}{2}-\frac{e^0}{2}\right]d\theta$$

$$\left(\frac{e^4-1}{2}\right)\theta\Bigg|_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}}$$

$$\left(\frac{e^4-1}{2}\right)(\pi)$$

ANS

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