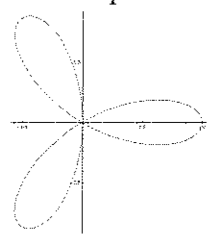
Lesson 34 Polar Regions II Board Sheet

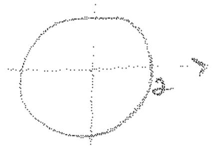
Use polar coordinates to find the volume of the given solid.

1) One loop of the rose $r = \cos(3\theta)$ = Area or Volume



$$\int_{\theta=-\frac{\pi}{6}}^{\theta=\frac{\pi}{6}} \int_{r=0}^{r=\cos(3\theta)} r\,dr\,d\theta \Rightarrow \text{area} = \frac{\pi}{12}$$

2) Under the cone $z = \sqrt{x^2+y^2}$ and above the disk $x^2+y^2 \leq 4$.

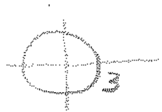


$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} \sqrt{r^2}\; r\,dr\,d\theta = \frac{16}{3}\pi$$

3) Below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the x-y plane.

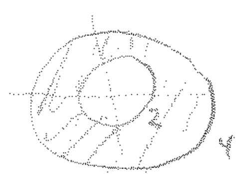
$z=0 \Rightarrow 18 = 2x^2+2y^2$

$9 = x^2+y^2$



$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=3} (18-(2r^2))\, r\,dr\,d\theta = 81\pi$$

4) Inside the sphere $x^2+y^2+z^2 = 16$ and outside the cylinder $x^2+y^2 = 4$.



$$2\int_{\theta=0}^{\theta=2\pi} \int_{r=2}^{r=4} \sqrt{16-r^2}\; r\,dr\,d\theta = 32\sqrt{3}\,\pi$$

5) A sphere of radius a

$$V = 2 \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=a} \sqrt{a^2-r^2}\; r\,dr\,d\theta = \frac{4}{3}\pi a^3$$

$z = \sqrt{x^2+y^2}$

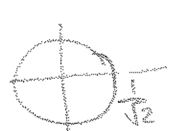
$z = \sqrt{1-(x^2+y^2)}$

intersect at

$x^2+y^2 = \frac{1}{2}$

$\sqrt{\frac{1}{2}}$

6) Above the cone $z = \overset{g(x,y)}{\sqrt{x^2+y^2}}$ and below the sphere $\overset{f(x,y)}{x^2+y^2+z^2} = 1$.



$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\frac{1}{\sqrt{2}}} (f(r,\theta) - g(r,\theta))\, r\,dr\,d\theta$$

$$(\sqrt{1-r^2} - r)\, r\,dr\,d\theta$$

$z = r$