Name: KEY

Section: ______________

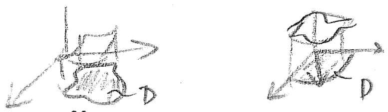
## MA205 Graded Homework #5 Polar and Rectangular Regions

**(40 points)**

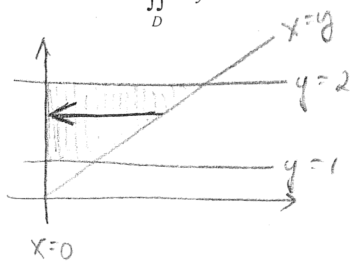
1) The last two lessons have dealt with general and polar ~~regions~~ coordinates. In your own words, describe what each is and provide a graphical sketch of each.

General ⇒ region of integration on x-y plane

Polar Coordinates ⇒ way of representing "some" general regions in alternate form from cartesian coordinates. $x = r\cos\theta$ $y = r\sin\theta$ $x^2+y^2=r^2$.




2) Evaluate $\iint_D x^2y^2\,dA$ where D is the figure bounded by $y=1,\ y=2,\ x=0,\ and\ x=y$.

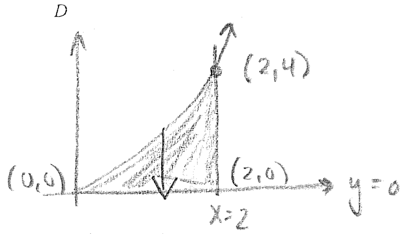



$$\int_{y=1}^{y=2}\int_{x=0}^{x=y} x^2y^2\,dx\,dy = \underline{\underline{3.5}}\ \text{ANS}$$

3) Evaluate the double integral. $D$ is bounded by $y=0$, $y=x^2$, and $x=2$.

$$\iint_D x\cos y\ dA,$$

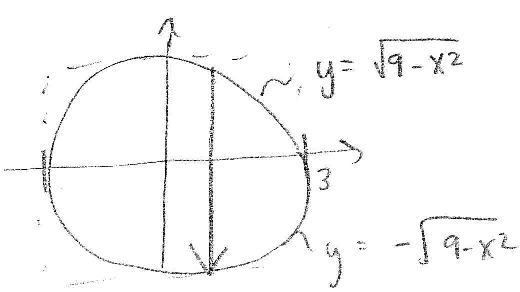
order does not matter,




$$\int_{x=0}^{x=2}\int_{y=0}^{y=x^2} x\cos(y)\,dy\,dx = \underline{\underline{.82}}\ \text{ANS}$$

4) Evaluate the double integral. $D$ is bounded by the circle with center the origin and radius 9.

$$\iint_D (5x-y)\ dA$$

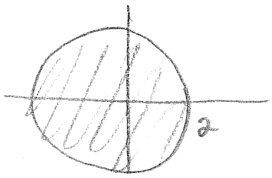



$$\int_{x=-9}^{x=9}\int_{y=-\sqrt{9^2-x^2}}^{y=\sqrt{9^2-x^2}} (5x-y)\,dy\,dx$$

$$\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=9} (5r\cos\theta - r\sin\theta)\,\boxed{r\,dr\,d\theta}$$

$$= \underline{0}\ \text{ANS}$$

5) Setup using polar coordinates to find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \le 4$. **Do not solve.** Hint: Draw the domain first.

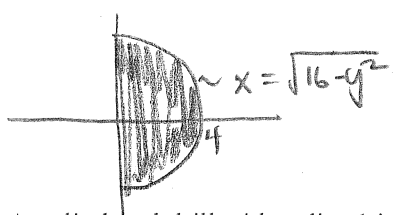



$$\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2} r^2 \boxed{r\,dr\,d\theta}$$

combine to get $r^3\,dr\,d\theta$

6) Setup the integral by changing to polar coordinates. $D$ is the region bounded by the semicircle $x = \sqrt{16 - y^2}$ and the $y$-axis. **Do not solve.** Hint: Draw the domain first.

$$\iint_D e^{-x^2-y^2}\,dA$$

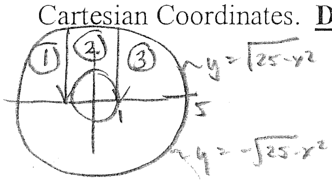



$$\int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}}\int_{r=0}^{r=4} e^{-r^2}\, r\,dr\,d\theta$$

Sphere
$x^2+y^2+z^2=5^2$
$f(x,y)=\pm\sqrt{25-x^2-y^2}$

7) A cylindrical drill with radius 1 is used to bore a hole through the center of a sphere of radius 5. Find the volume of the ring-shaped solid that remains. Setup the integral in **both** Polar and Cartesian Coordinates. **Do not solve.** Hint: Draw the domain first.

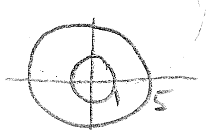
Cartesian




$$4\left[\int_{x=-5}^{x=-1}\int_{y=0}^{y=\sqrt{25-y^2}} f(x,y)\,dy\,dx + \int_{x=-1}^{x=1}\int_{y=\sqrt{1-x^2}}^{y=\sqrt{25-y^2}} f(x,y)\,dy\,dx + \int_{x=1}^{x=5}\int_{y=0}^{y=\sqrt{25-x^2}} f(x,y)\,dy\,dx\right]$$

(1) (2) (3)

Polar



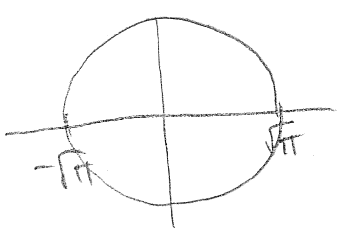

$$\underline{\underline{2}}\int_{\theta=0}^{\theta=2\pi}\int_{r=1}^{r=5}\sqrt{25-r^2}\,\boxed{r\,dr\,d\theta}$$

MULTIPLY BY 2 FOR TOP + BOTTOM CARTESIAN + POLAR

8) You choose Polar or Cartesian Coordinates, but you must solve by hand. Hint: Draw the domain first.

$$\int_{-\sqrt{\pi}}^{\sqrt{\pi}}\int_{-\sqrt{\pi-y^2}}^{\sqrt{\pi-y^2}} \sin(x^2+y^2)\,dx\,dy$$

POLAR MUCH EASIER!




$$\Rightarrow \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=\sqrt{\pi}} \sin(r^2)\,\boxed{r\,dr\,d\theta}$$

$u = r^2$ $\quad du = 2r\,dr$ $\quad dr = \frac{du}{2r}$

$$\frac{\sin(u)}{2} \Rightarrow -\frac{\cos(r^2)}{2}\Big|_{r=0}^{r=\sqrt{\pi}}$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$\int_{\theta=0}^{\theta=2\pi} d\theta$$

$$\underline{\underline{2\pi}}$$