Graded Homework Due 7 November in Class

For the following problems develop a Differential Equation and solve it using the following solution types:

- **Numerical Solution:** Using initial conditions, use Eulers Method with StepSize=1 to approximate value of the function after 3 iterations (t=3 units), graph results by hand. Then use mathematica code developed in class and step size of .01 to answer the question.
- **Graphical Solution:** Using initial conditions solve the problem using slope fields (Recall command DEPlot in Mathematica), comment on the long term behavior of the function (is there a limiting value?).
- **Analytical Solution:** Using initial conditions, solve the Differential Equation analytically by finding the particular solution (confirm with Mathematica DSolve), if the DE is separable you must show all work (by hand).

Be sure to define your variables-recall the whole purpose of a DE is to model a real-world phenomenon through rates (differential elements) to find the underlying function (dependent variable).

Question 1: Newton's Law of Cooling

After spending many hours studying for your math class you decide to reward yourself with a case of Stiffler Soda. Unfortunately Grant Hall only had a warm case left, the instructors got there first and got all of the cold ones. You buy several bags of ice and decide to chill it at home. When you get home the temperature of the soda is 76 degrees F and you proceed to pack it in ice. After ten minutes the temperature has cooled to 70 degrees F. If Stiffler Soda is best served at 60 degrees F or cooler, how much <u>longer</u> must you wait to enjoy your Sriffler Soda? Hint: you must make an assumption IOT solve.

Question 2: Population Growth

a) Koala bears live on a farmer's land in Australia. The population's rate of growth is proportional to the population of Koala Bears. The initial population is 100 and after one year 120 koalas are present. Use this to find out how long it will take the population to increase from 100 to 300 koalas.
b) Consider: Once the population reaches 300, a Koala-eating anacando begins eating koalas, at a rate of 80 koalas every year. How long will it take until all the koalas are gone? Modify your DE to answer this question (setup DE Only do not solve).

Question 3: Mixing Problem

A tank contains 100 gallons of salt water which contains 10 lbs of salt. A salt solution of 2 lbs salt per gallon enters the tank at a rate of 3 gallons per minute while a flow of fresh water runs into the tank at rate 4 gallons per minute. The well-stirred solution runs out of the tank at rate of 7 gal/min.

(1) NEWTON'S LAW OF COOLING

Assume • fridge $\Rightarrow$ 32°F
• Soda doesn't change temp from store to home.

$$\frac{dT}{dt} = k(T - T_s)$$

$$\frac{dT}{dt} = k(T - 32)$$

$T(0) = 76$

$T(10) = 70$

$$\frac{dT}{dt} = -\frac{6}{10} = -.6$$

use $\frac{dT}{dt} = k(T - 32)$ w/ initial conditions.

$-.6 = k(76 - 32)$ $\quad k = -.0137$

$-.6 = k(70 - 32)$ $\quad k = -.0158$

$\bar{k} = -.0148$

So $\frac{dT}{dt} = -.0148(T - 32)$

Slope Field = 30 min $\Rightarrow$ So wait 20 more minute

Separable? Yes

$$\int \frac{1}{(T-32)}\,dT = \int -.0148\,dt$$

$$\ln(T-32) = -.0148t + C$$

$$T(t) = Ce^{-.0148t} + 32 \quad C = 44$$

$$T(t) = 44e^{-.0148t} + 32$$

$$60 = 44e^{-.0148t} + 32$$

$$t = \underline{30.15}$$

So must wait additional 20.15 min

6 2)

$$\frac{dP}{dt} = kP$$

$P(0) = 100$

$P(1) = 120$

$$\frac{dP}{dt} = \frac{20}{1} = 20$$

$20 = k(100)$ $\quad k = .2$ $\quad \bar{k} = .185$

$20 = k(100)$ $\quad k = .17$

$$\frac{dP}{dt} = .185P$$

$P(6t \approx 6$ years

Separate variable

$P(t) = P_0 e^{kt}$

$\cancel{300} = \cancel{100} e^{.185t}$

$3 = e^{.185t}$

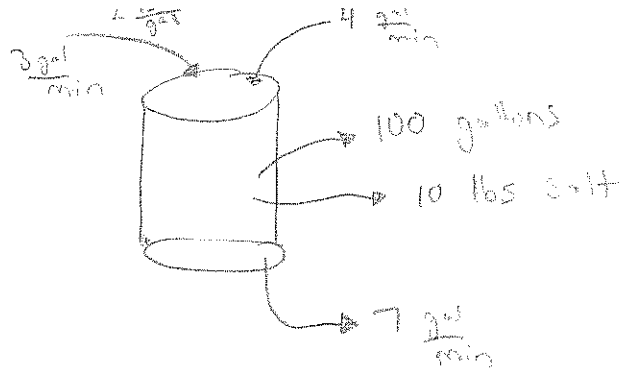
$t = \underline{\underline{5.73}}$ years

b) $\frac{dP}{dt} = kP - c$

$$\frac{dP}{dt} = kP - 80$$

* The catch is $P(0) = 300$ or the time starts when $P(t) = 300$.

3)



$\boxed{\text{Mixing Problem}}$

$C(t)$ = Concentration of Salt in the tank at time t

t = time in minutes

$$\frac{dC}{dt} = (\text{Rate in}) - (\text{Rate out})$$

* where the Rate is the amount of Salt per minute

$$\text{Rate in} = 3\,\frac{\cancel{gal}}{min} \times 2\,\frac{lbs\ salt}{\cancel{gal}} + 4\,\frac{gal}{min} \times 0\,\frac{lbs\ salt}{gal}$$

$$= 6\,\frac{lbs}{min}$$

$$\text{Rate out} = 7\,\frac{\cancel{gal}}{min} \times \frac{C(t)\ lbs\ salt}{100\ \cancel{gallons}}$$

$$= 0.07\,C(t)\,\frac{lbs}{min}$$

$$\boxed{\frac{dC}{dt} = 6 - .07\,C(t)} \Rightarrow DE$$

- How would you solve for $C(t)$ after 5 mins?

I would solve the above DE using Separation of Variables. I would then use the initial Condition of $C(0) = 10$ to solve for the constant of integration. I would then use $t = 5$ to solve for $C(5)$.

Lesson 40 Euler's Method Worksheet

- Must have initial conditions
- "Integration long hand" Old School Method (Thank goodness we've matured…)
- Quality of approximation based on step size
- Use technology to increase quality of approximation, step size goes to 0.

Newton's Law of Cooling

Differential equation(book defines as F(x,y): $\frac{dT}{dt} = -.0148\,(T-32)$

Initial Conditions(ex: x(0), y(0)): $T(0) = 76 \quad T(1) = 76$

Step Size(book defines as *h*): 1

End criteria (Approximate Solution, # Steps, or desired dependent variable value ex: y(x)=C)

i (iteration)	X_{old} t	Y_{old} T	Step Size*(DE(x_{old},y_{old}) 1	X_{new}	Y_{new}
1	0 Initial Cond	76 Initial Cond	-.6512	1	75.32
2	1	75.32	-.6411	2	74.67
3	2	74.67	-.6315	3	74.03
4					
5					

*Note: For iteration 2, X_{new} & Y_{new} from iteration 1, become X_{old} & Y_{old} for iteration 2.

Population Growth

Differential equation(book defines as F(x,y): $\frac{dP}{dt} = .185P$

Initial Conditions(ex: x(0), y(0)): $P(0) = 100$

Step Size(book defines as *h*): 1

End criteria (Approximate Solution, # Steps, or desired dependent variable value ex: y(x)=C)

i (iteration)	X_{old} t	Y_{old} P	Step Size*(DE(x_{old},y_{old})	X_{new}	Y_{new}
1	0 Initial Cond	100 Initial Cond	18.5	1	118.5
2	1	118.5	21.9	2	140.4
3	2	140.5	26	3	166.4
4					
5					

*Note: For iteration 2, X_{new} & Y_{new} from iteration 1, become X_{old} & Y_{old} for iteration 2.

Graded Homework #6

Question 1 Newton' s Law of Cooling

Numerical Solution (Euler' s Method)

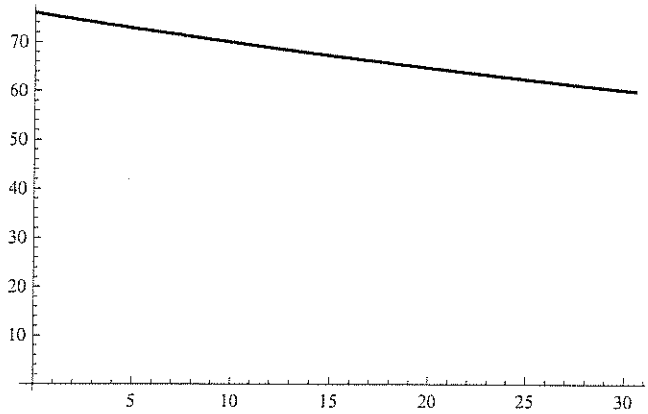
```
Clear[x, y, step, iterations];
slope[x_, y_] = -.0148 * (y - 32); (*define the differential equation*)
step = .01; (*define step size*)
x[0] = 0; (*define initial x value*)
y[0] = 76; (*define initial y value*)
iterations = 3050;

Do[
 x[i] = x[i - 1] + step;
 y[i] = y[i - 1] + step * slope[x[i - 1], y[i - 1]],
 {i, 1, iterations}]
Print["Euler's Method Values"]
values = TableForm[Table[{x[i], y[i]}, {i, iterations - 10, iterations}],
  TableHeadings → {None, {"x", "y"}}]
eulersoln = ListPlot[Table[{x[i], y[i]}, {i, 1, iterations}], Joined → True,
  PlotStyle → {{RGBColor[0, 0, 1], Thickness[0.005]}}, AxesOrigin → {0, 0}]
```

Euler's Method Values

Out[172]//TableForm=

x	y
30.4	60.0569
30.41	60.0528
30.42	60.0486
30.43	60.0445
30.44	60.0403
30.45	60.0362
30.46	60.032
30.47	60.0279
30.48	60.0237
30.49	60.0196
30.5	60.0155

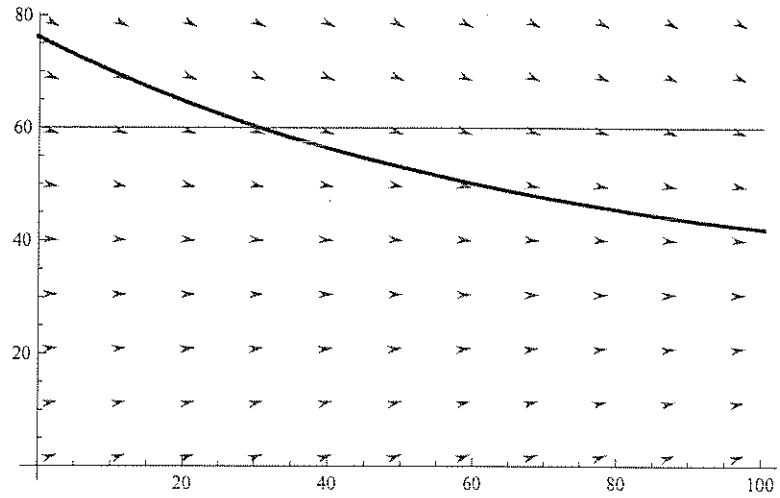
Out[173]=



Graphical Solution-Slope Field

In[174]:=
```
Show[DEPlot[-.0148 * (T - 32), {t, 0, 100}, {T, 0, 80},
  InitialPoints → {{0, 76}, {10, 70}}, AxesOrigin → {0, 0}], Plot[60, {t, 0, 100}]]
```

Out[174]=



Analytical Solution (Verify Work by Hand)-Separation of Variables

```
Clear[b];
```

```
DSolve[{-.0148 * (b[t] - 32) == b'[t], b[0] == 76}, b[t], t]
```

$\{\{b[t] \to e^{-0.0148\,t}\,(44. + 32.\,e^{0.0148\,t})\}\}$

```
NSolve[e^(-0.0148`t) (44.` + 32.` e^(0.0148`t)) == 60, t]
```

$\{\{t \to 30.5395\}\}$

**Notice how all three solution methods provide similar results!

```
DEPlot[.185 p, {t, 0, 10}, {p, 0, 500},
 InitialPoints → {{0, 100}, {1, 120}}, AxesOrigin → {0, 0}]
```

Question 2 Population Growth

Numerical Solution (Euler' s Method)

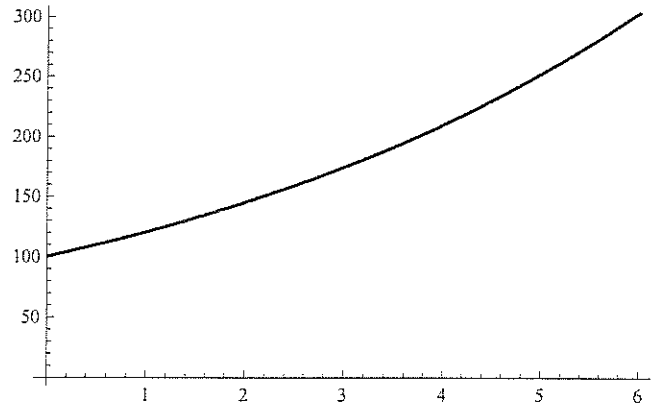
In[183]:=
```
Clear[x, y, step, iterations];
slope[x_, y_] = .185 y; (*define the differential equation*)
step = .01; (*define step size*)
x[0] = 0; (*define initial x value*)
y[0] = 100; (*define initial y value*)
iterations = 600;

Do[
 x[i] = x[i - 1] + step;
 y[i] = y[i - 1] + step * slope[x[i - 1], y[i - 1]],
 {i, 1, iterations}]
Print["Euler's Method Values"]
values = TableForm[Table[{x[i], y[i]}, {i, iterations - 10, iterations}],
  TableHeadings → {None, {"x", "y"}}]
eulersoln = ListPlot[Table[{x[i], y[i]}, {i, 1, iterations}], Joined → True,
  PlotStyle → {{RGBColor[0, 0, 1], Thickness[0.005]}}, AxesOrigin → {0, 0}]
```

Euler's Method Values

Out[187]//TableForm=

x	y
5.9	297.574
5.91	298.124
5.92	298.676
5.93	299.228
5.94	299.782
5.95	300.336
5.96	300.892
5.97	301.449
5.98	302.006
5.99	302.565
6.	303.125

Out[188]=



Graphical Solution-Slope Field

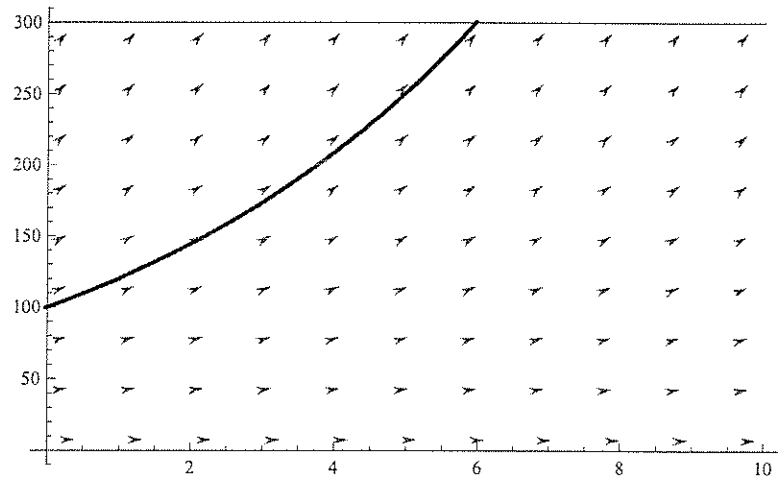
In[191]:=
```
Show[DEPlot[.185 P, {t, 0, 10}, {P, 0, 300},
  InitialPoints → {{0, 100}, {1, 120}}, AxesOrigin → {0, 0}], Plot[300, {t, 0, 10}]]
```

Part::partw : Part 2 of
{{{}, ≪1≫, {Hue[0.67, 0.6, 0.6], ≪9≫[≪6≫], Line[{{0., 100.}, {0.00182545, 100.034}, {0.00365089, 100.068}, ≪5≫, {0.243484, 104.607}, {0.361746, 106.921}, ≪68≫}]}}} does not exist. ≫

Part::partw : Part 2 of
{{{}, ≪1≫, {Hue[0.67, 0.6, 0.6], ≪9≫[≪6≫], Line[{{0., 100.}, {0.00182545, 100.034}, {0.00365089, 100.068}, ≪5≫, {0.243484, 104.607}, {0.361746, 106.921}, ≪68≫}]}}} does not exist. ≫

Part::partw : Part 2 of
{{{}, ≪1≫, {Hue[0.67, 0.6, 0.6], Thickness[0.005], Line[{{1., 120.}, {1.00152, 120.034}, {1.00305, 120.068}, ≪5≫, {1.20317, 124.596}, {1.30185, 126.892}, ≪68≫}]}}} does not exist. ≫

General::stop : Further output of Part::partw will be suppressed during this calculation. ≫

Out[191]=


Analytical Solution (Verify Work by Hand)-Separation of Variables

```
Clear[p];
```

In[192]:=
```
Simplify[DSolve[{.185 * (p[t]) == p'[t], p[0] == 100}, p[t], t]]
```

Out[192]= $\{\{p[t] \to 100\, e^{0.185\, t}\}\}$

$\text{Solve}\left[100\, e^{0.185`\, t} == 300,\ t\right]$

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

Out[199]= {{t → 5.93844}}

**Notice how all three solution methods provide similar results!