Name: KEY
Section: _____

MA205 Quiz 2 (3-D Estimations)

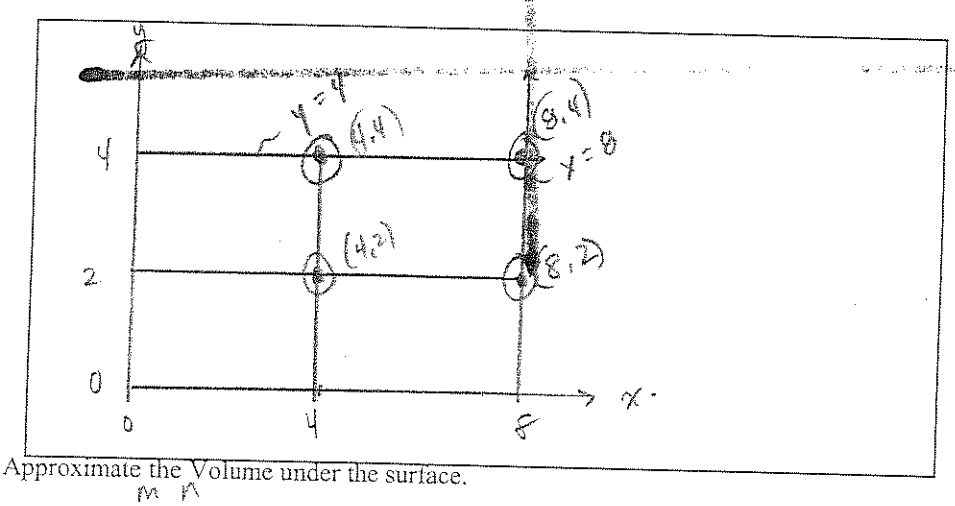
20 points

Find an approximation for the integral.

$$\iint_R (4x - 5y^2)\ dA$$

Use a double Riemann sum with $m = 2$ $n = 2$ and the sample point in the upper right corner to approximate the double integral, where $R = \{ (x, y) \mid 0 \le x \le 8,\ 0 \le y \le 4 \}$.

Draw the Domain.



Approximate the Volume under the surface.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_j)\,\Delta x\,\Delta y \qquad \Delta x = \frac{8-0}{2} = 4$$

$$\Delta y = \frac{4-0}{2} = 2$$

$$8\left[f(4,2) + f(4,4) + f(8,4) + f(8,2) \right]$$

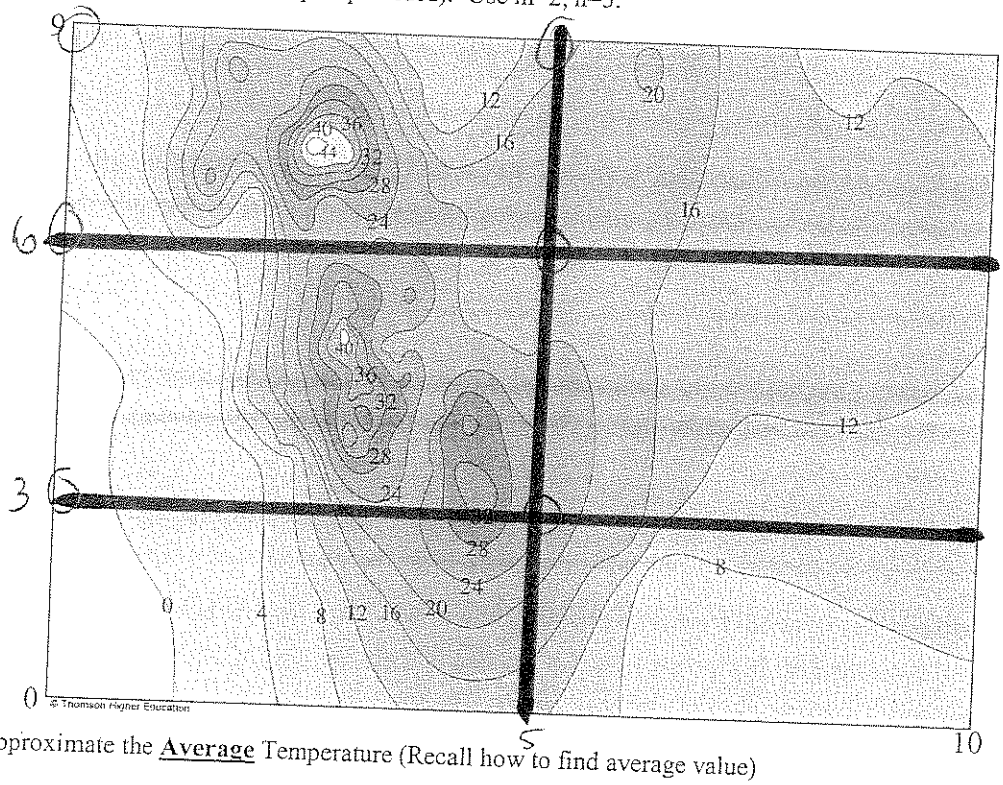
$$8\left[(-4) + (-64) + (12) + (-48) \right] \Delta x \Delta y = 8$$

$$832 \text{ UNITS}^3 \quad (\text{BELOW } z = 0)$$

How is the approximation (Riemann sum) related to the integral depicted above? Provide a relationship.

$$\lim_{m,n \to \infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_i)\,\Delta x\,\Delta y = \int_0^8 \int_0^4 (4x - 5y^2)\,dy\,dx$$

2. Use an upper left approximation to estimate the average temperature of West Point at 1400 yesterday afternoon from the below contour plot where units are in degrees F. Use R[0,10]x R[0,9] units are in miles. Show all work (use the plot provided). Use m=2, n=3.



Approximate the **Average** Temperature (Recall how to find average value)

$\sum f(x_i, y_j)\,\Delta x\,\Delta y$

$15\left[f(0,3)+f(0,6)+f(0,9)+f(5,3)+f(5,6)+f(5,9)\right]$

$15\left[6+0+(-2)+(14)+18+24\right]$

900

$\Delta x = \frac{10-0}{2} = 5$

$\Delta y = \frac{9-0}{3} = 3$

$\Delta x \Delta y = 15$

$\text{AVG} = \frac{\text{VOL}}{\text{AREA}} = \frac{900}{90} = 10^{\circ}$

$\pm 2^{\circ}$ ACCEPTABLE

Comment on the quality of your approximation.

Under Approximation → does not account for high values.

Name: MAKE UP

Section: _____

MA205 Quiz 2 (Estimations)

20 Points

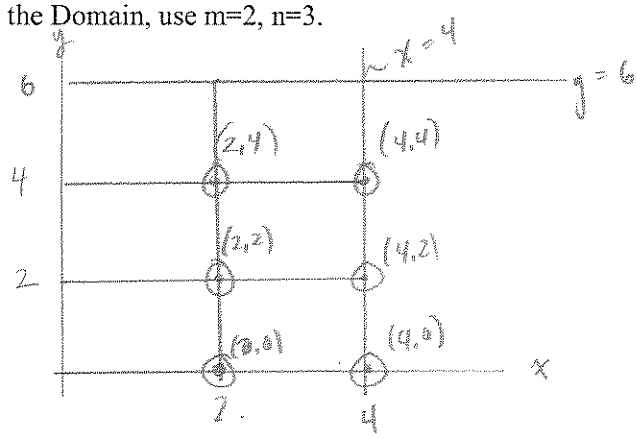
1. Given the integral:

$$\iint_R (x+y)dA \text{ where } R = \{(x,y) | 0 \le x \le 4, 0 \le y \le 6\}$$

a. Sketch the Domain, use m=2, n=3.

$\Delta x = \frac{4-0}{2} = 2$

$\Delta y = \frac{6-0}{3} = 2$



b. Estimate the value of the integral using a bottom right corner approximation.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_{j-1})\,\Delta x\,\Delta y$$

$\Delta x \Delta y = 4$

$$= 4\left[f(2,0) + f(2,2) + f(2,4) + f(4,0) + f(4,2) + f(4,4)\right]$$

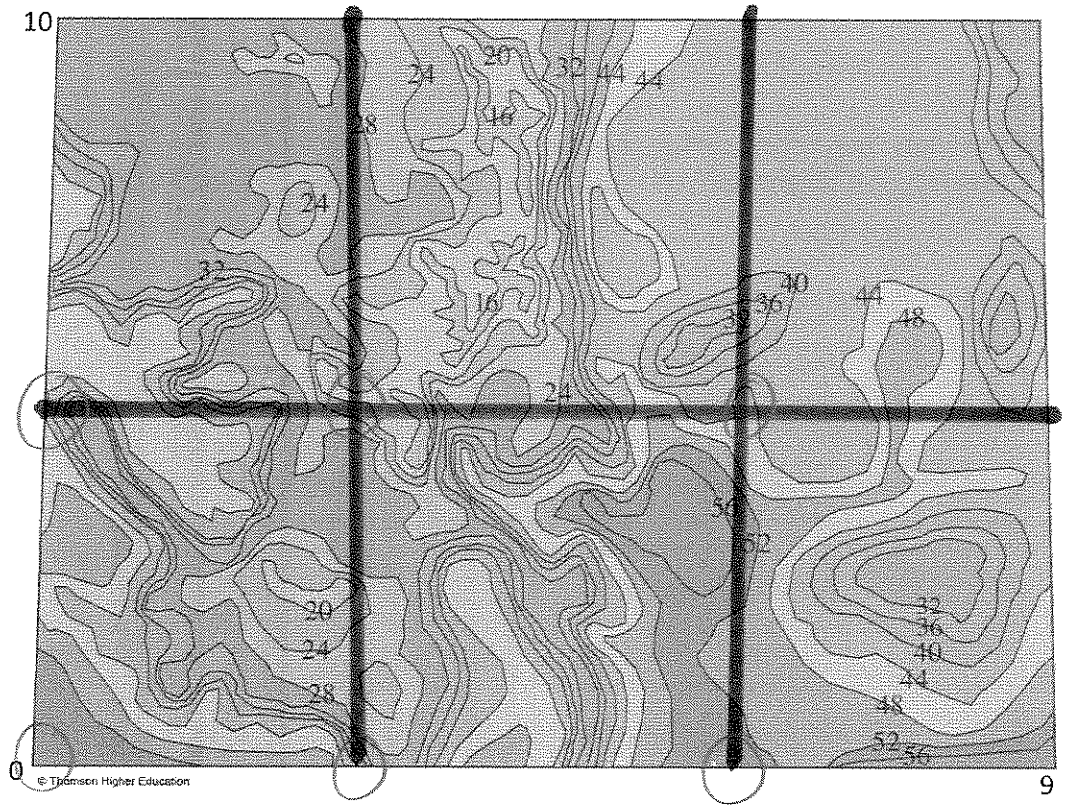
$$= 4\left[2 + 4 + 6 + 4 + 6 + 8\right]$$

$$= 120 \text{ units}^3$$

c. Provide a mathematical relationship between the approximation (bottom right corner) and the given integral.

$$\lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_{j-1})\,\Delta x\,\Delta y = \int_0^6 \int_0^4 x+y \; dx\,dy$$

2. Use a bottom left approximation to estimate the average temperature of West Point at 0600 yesterday morning from the below contour plot where units are in degrees F. Use (x,y) $\in$ R[0,9]x R[0,10] units are in miles. Show all work (use the plot provided). Use m=3, n=2.



a. Approximate the **Average** Temperature (Recall how to find average value)

$\Delta x \Delta y = (3)(5) = 15$

$15\left[f(0,0) + f(0,5) + f(3,0) + f(3,5) + f(6,0) + f(6,5)\right]$

$15\left[40 + 36 + 32 + 24 + 54 + 44\right]$

° ESTIMATE TEMP FROM CONTOUR PLOT

$3450 \Rightarrow$ DIVIDE BY $\text{AREA}_{\text{DOMAIN}}$ $\quad \frac{3450}{9(10)} = 38^{\circ}F$

b. Comment on the quality of your approximation.

DOES NOT APPEAR TO CAPTURE MAX TEMPS

SO THIS MAY BE AN UNDER APPROX.