MA104 Course Homework # 1
Due 31 January 2008

Instructions: Complete all of the problems below. Explain which derivative rules are being applied, as appropriate. Show all necessary steps when taking derivatives (do not rely on Mathematica to take derivatives for you). You are encouraged to work in groups to complete this assignment. Document all assistance in accordance with the Dean's Document of Written Work (DWW). Neatness and organization will be graded.

1. Given $f(x) = x^3 + 2x^2 - x + 1$, find $f'(x), f''(x), f'''(x)$, and $f''''(x)$. Graph all 5 functions on the same axes (using Mathematica) on the domain $\{-10 \leq x \leq 10\}$.

   Example:

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Plot[{x^2, 2 x - 1, 2}, {x, -2, 2}]
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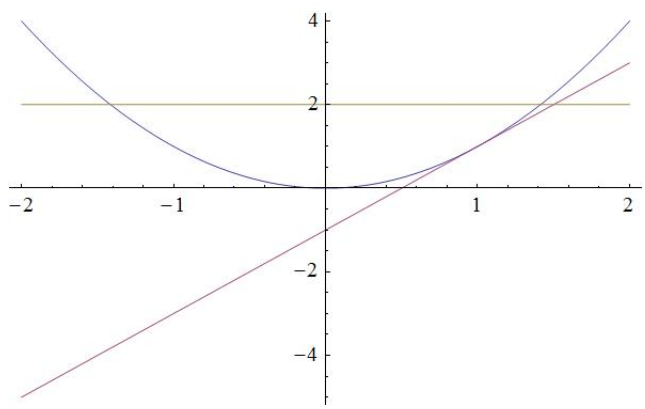

2. The curve $y = \dfrac{x}{1+x^2}$ is called a serpentine.

   (a) Find an equation of the tangent line to the curve at the point $(3, .3)$.
   (b) Plot the curve and the tangent line on the same axes on the domain $\{-10 \leq x \leq 10\}$.
   (c) Find y', and y''.
   (d) Plot y, y', and y'' on the same axes on the domain $\{-10 \leq x \leq 10\}$.

3. Assume that $f(x)$ is a differentiable function. Find $\dfrac{dy}{dx}$ for each of the functions below.

   (a) $y = x^2 f(x)$
   (b) $y = \dfrac{f(x)}{x^2}$
   (c) $y = \dfrac{x^2}{f(x)}$
   (d) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

4. If $g(x)$ is differentiable, the **Reciprocal Rule** says that:

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}.$$

(a) Use the Quotient Rule to prove the Reciprocal Rule.

(b) Use the Reciprocal Rule to differentiate $y(s) = \dfrac{1}{s + 2e^s}$.

(c) Use the Reciprocal Rule to verify that the Power Rule is valid for negative integer exponents, that is,

$$\frac{d}{dx}[x^{-n}] = -nx^{-n-1}, \quad n \in 1, 2, 3, ...$$