MA205 Lesson 16

Probability and the Integral

Thursday, September 3, 2007

Outline

1. From Lesson 15
   - Work
2. Probability
   - Probability and the Integral
3. Look at for Next Time

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Work on Fluids

- How much work is required to syphon beer out of a carboy that is 20" high and has a diameter of 10.5" with beer up to 13" in the carboy. Where this beer has a specific gravity of 1.020 and weighs 63.75 lbs per cubic foot.
  - Answer 46.72 ft-lb, for weight of 60 lbs per cubic foot the answer is 43.97 ft-lb
- How much work does the CO2 bottle do to dispense beer out of a keg that is 26" high and has an 8" diameter with beer up to 24". Where this beer has a specific gravity of 1.020 and weighs 63.75 lbs per cubic foot.
  - Answer 51.92 ft-lb, for weight of 60 lbs per cubic foot the answer is 49.87

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Probability and the Integral

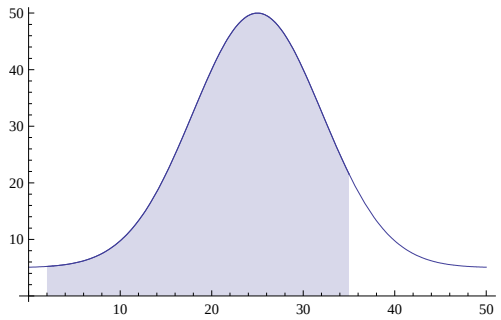
1. Continuous Random Variables: Quantities whose values range over an interval of real numbers. For example the lifetime of a randomly chosen battery.
2. Probability: The probability that our randomly chosen battery will last between 100 and 200 hours would be written as $P(100 \leq X \leq 200)$.
3. The probability of this battery lasting between 100 and 200 will have a value between 0 and 1.
4. Every continuous random variable X has a **probability density function** f or **PDF**. This means that the probability that X lies between a and b is found by integrating f from a to b.

Probability and the Integral

1. So this integral looks like:

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

2. Looking at the following graph if we wanted to know the probability of it raining between 2 and 35 inches a year we would find it is equal to the area under the curve between 2 and 35.



Probability and the Integral

1. Generaly the PDF f of a random variable X satisfies the condition that $f \geq 0$ for all x. Because probability is measured from 0 to 1, the integral must meet the following:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

2. The first step of using integration in evaluating a PDF is to verify it meets the above.
3. Next we can find the probability of say the rain being between 15 and 20 inches $P(15 \leq X \leq 20)$ by integrating $f(x)$ from 15 to 20.

Probability and the Integral

1. Next we might want to know the average rain fall over the last 10 years. Again we will use integration. We will assume it rains at least 1 inch per year and no more than 300 inches.

$$\int_{1}^{300} x\, f(x)dx = \mu$$

2. μ is known as the mean rain fall.

Practical Exercise

1. Major Bowman is known for going over the speed limit on post in his 1965 Mustang. However, he never goes more than 5 mph over the speed limit. Let $X =$ the speed over the speed limit he travels. Suppose the pdf of X is:

$$f(x) = kx^2 \quad 0 \geq X \geq 5, \quad f(x) = 0 \quad X < 0$$

2. μ is known as the mean rain fall.

Probability of getting a ticket

- Look at the course guide
- Stewart Chapter 8, Section 5, PG 575-581.

Questions