MA205

Board Sheet - 25 September 2007, Estimations Using Data, Lesson 21-22

How to Estimate Using Data and Functions

1. $R = [a,b]X[c,d] = \{(x,y) \in \mathbf{R}^2 | a \le x \le b, c \le y \le d\}$
2. $S = \{(x,y,z) \in \mathbf{R}^3 | 0 \le z \le f(x,y), (x,y) \in R\}$
3. Now we make rectangles each with area $\Delta A = \Delta x \Delta y$.
4. We chose sample points and get $f(x_{ij}^*, y_{ij}^*)\Delta A$. Where our sample point could be the midpoint, upper-left corner, upper-right corner, lower-left corner, or lower-right corner of each rectangle.
5. Next we set up Riemann Sums $V = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$
6. Finally we get the double integral $\iint_R f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$
7. Midpoint Rule for Double Integrals $\iint_R f(x,y)dA \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(\bar{x_i}, \bar{y_i})\Delta A$ Where $\bar{x_i}$ is the midpoint of $[x_{i-1}, x_i]$ and $\bar{y_i}$ is the midpoint of $[y_{j-1}, y_j]$.
8. Average Value $f_{ave} = \dfrac{1}{A(R)} \iint_R f(x,y)dA$

Mathematica for some problems:

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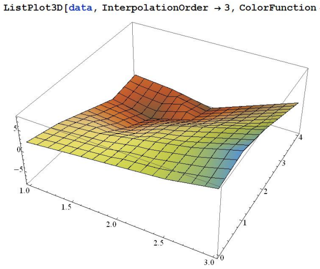
```
(*  Here is an estimate of what the graph of the table of values
 for Lesson 21 Mechanics Based Problems #1 may look like  *)

data = List[{1, 0, 2}, {1, 1, 0}, {1, 2, -3}, {1, 3, -6}, {1, 4, -5},
   {1.5, 0, 3}, {1.5, 1, 1}, {1.5, 2, -4}, {1.5, 3, -8}, {1.5, 4, -6},
   {2, 0, 4}, {2, 1, 3}, {2, 2, 0}, {2, 3, -5}, {2, 4, -8}, {2.5, 0, 5},
   {2.5, 1, 5}, {2.5, 2, 3}, {2.5, 3, -1}, {2.5, 4, -4}, {3, 0, 7},
   {3, 1, 8}, {3, 2, 6}, {3, 3, 3}, {3, 4, 0}];

ListPlot3D[data, InterpolationOrder → 3, ColorFunction → "SouthwestColors"]
```




```
(* a use midpoint and four regions *)
(* break the table up into four regions where each box is 2 by 1 *)
(* look at the value in the middle of that box and mult by area *)
(* MVP = 2*1*(data(1.5,1)+data(1.5,3)+data(2.5,1)+data(2.5,3)) *)
MVP = 2 * 1 (1 + (-8) + 5 + (-1))

-6
```

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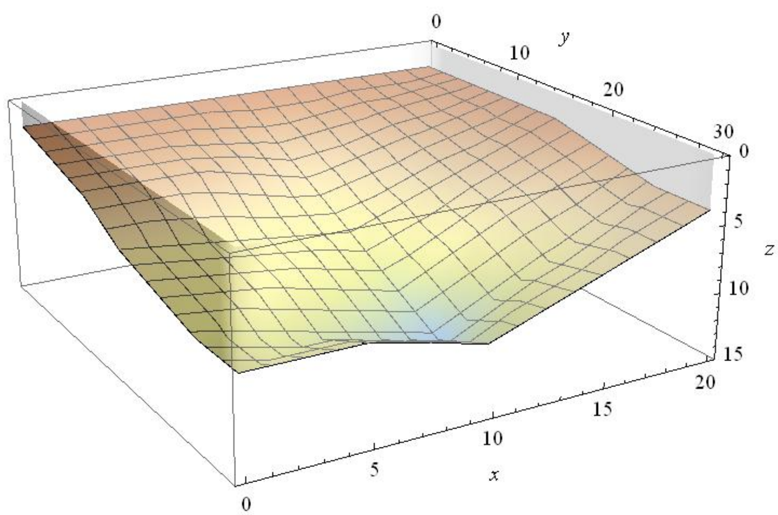
```
In[68]:= swimmingpool = List[{0, 0, 2}, {0, 5, 3}, {0, 10, 4}, {0, 15, 6},
            {0, 20, 7}, {0, 25, 8}, {0, 30, 8}, {5, 0, 2}, {5, 5, 3},
            {5, 10, 4}, {5, 15, 7}, {5, 20, 8}, {5, 25, 10}, {5, 30, 8},
            {10, 0, 2}, {10, 5, 4}, {10, 10, 6}, {10, 15, 8}, {10, 20, 10},
            {10, 25, 12}, {10, 30, 10}, {15, 0, 2}, {15, 5, 3}, {15, 10, 4},
            {15, 15, 5}, {15, 20, 6}, {15, 25, 8}, {15, 30, 7}, {20, 0, 2},
            {20, 5, 2}, {20, 10, 2}, {20, 15, 2}, {20, 20, 3}, {20, 25, 4},
            {20, 30, 4}];
         (* To get the volume one would find the area of each box
          from the table then multiply by the depth in that box *)
         (* so 5*5(depth(0,0)+depth(0,5)+...+depth(20,30)) *)
         ListPlot3D[swimmingpool, InterpolationOrder → 3,
          ColorFunction → "SouthwestColors", AxesLabel → {x, y, z},
          PlotRange → {0, 15}, Filling → Bottom]
         (* If we knew the Formula for the pool we could get the
          exact volume *)
         (* We would use the double integral of the function *)
```

Out[69]=

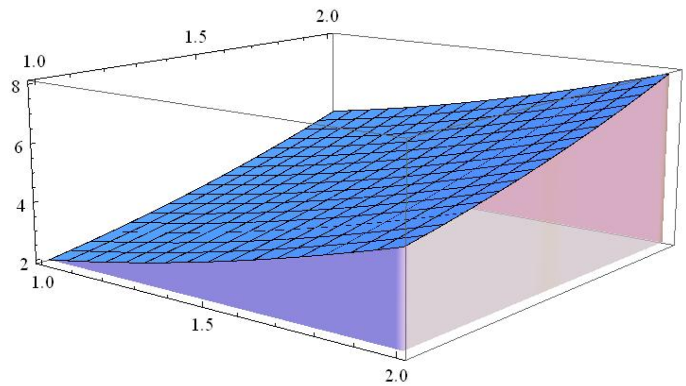


- Lesson 21 Classroom Example

```
(* Find the volume of the surface z=x^2+y^2 *)
(* Use m=n=2 and m=n=4 then find the exact *)

f[x_, y_] := x^2 + y^2

Plot3D[f[x, y], {x, 1, 2}, {y, 1, 2}, Filling → Bottom]
```



$$\int_1^2 \int_1^2 f[x, y]\, dx\, dy$$

$$\frac{14}{3}$$

$$\int \int f[x, y]\, dx\, dy$$

$$\frac{1}{3} x y \left(x^2 + y^2\right)$$

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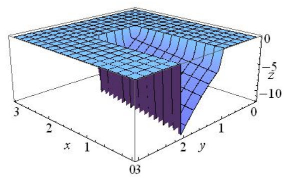
In[45]:=
```
bullet[x_, y_] := x - 3 y^2;
```

In[46]:=
```
Plot3D[If[0 ≤ x ≤ 2 && 1 ≤ y ≤ 2, bullet[x, y], 0], {x, 0, 3},
  {y, 0, 3}, ViewPoint → {-2, 2, 1}, AxesLabel → {x, y, z}]
```

Out[46]=



In[47]:=
```
a = 0; b = 2; c = 1; d = 2;
```

$$\texttt{deltax} = \frac{b-a}{n};$$

$$\texttt{deltay} = \frac{d-c}{m};$$

```
deltaa = deltax * deltay;
x[i_] = a + i * deltax;
y[j_] = c + j * deltay;
```

$$\texttt{lowerleft}[m\_, n\_] = \sum_{i=1}^{n}\sum_{j=1}^{m} \texttt{bullet}[x[i-1], y[j-1]] * \texttt{deltaa};$$

$$\texttt{lowerright}[m\_, n\_] = \sum_{i=1}^{n}\sum_{j=1}^{m} \texttt{bullet}[x[i], y[j-1]] * \texttt{deltaa};$$

$$\texttt{upperleft}[m\_, n\_] = \sum_{i=1}^{n}\sum_{j=1}^{m} \texttt{bullet}[x[i-1], y[j]] * \texttt{deltaa};$$

$$\texttt{upperright}[m\_, n\_] = \sum_{i=1}^{n}\sum_{j=1}^{m} \texttt{bullet}[x[i], y[j]] * \texttt{deltaa};$$

$$\texttt{mid}[m\_, n\_] = \sum_{i=1}^{n}\sum_{j=1}^{m} \texttt{bullet}\left[\left(\frac{x[i-1]+x[i]}{2}\right), \left(\frac{y[j]+y[j-1]}{2}\right)\right] * \texttt{deltaa};$$

$$\texttt{exact} = \int_1^2\int_0^2 \texttt{bullet}[x, y]\, dx\, dy;$$

In[59]:=
```
nn = 4;
mm = 4;
```

In[61]:=
```
lowerleft[nn, mm] // N
lowerright[nn, mm] // N
upperright[nn, mm] // N
upperleft[nn, mm] // N
mid[nn, mm] // N
exact // N
```

Out[61]=
-10.3125

Out[62]=
-9.3125

Out[63]=
-13.8125

Out[64]=
-14.8125

Out[65]=
-11.9688

Out[66]=
-12.

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```
In[74]:= z1[x_, y_] := 50 - x^2 - y^2

In[75]:= z2[x_, y_] := x^2 + y^2
```

In[76]:= $\int_{-5}^{5}\int_{-5}^{5}(z1[x, y] - z2[x, y])\, dx\, dy$ // N

```
Out[76]=
        1666.67

        (* to get the volume between the two surfaces you
         would do the above integral *)
```

(* to estimate it you would have to use dA=$\pi(r_i - r_{i-1})^2$ *)

```
        (* then eval z1 and z2 at a point between the two
         radius *)
```

(* Finally multiply dA*($z1_{ri}$-$z2_{ri}$) for as many rings you divide the region into *)

```
In[85]:= plot1 = Plot3D[z1[x, y], {x, -5, 5}, {y, -5, 5}];

In[86]:= plot2 = Plot3D[z2[x, y], {x, -5, 5}, {y, -5, 5}];

In[87]:= Show[plot1, plot2]

Out[87]=
```