

MA205

Board Sheet - 04 October 2007, Polar Coordinates

Step by Step COM

1. Draw and label the region and look at the density function $\rho(x, y)$

2. Compute Mass where $m = \iint_D \rho(x, y) dA$

3. Compute the Moments for x and y where

$$M_x = \iint_D y\rho(x, y) dA \quad M_y = \iint_D x\rho(x, y) dA$$

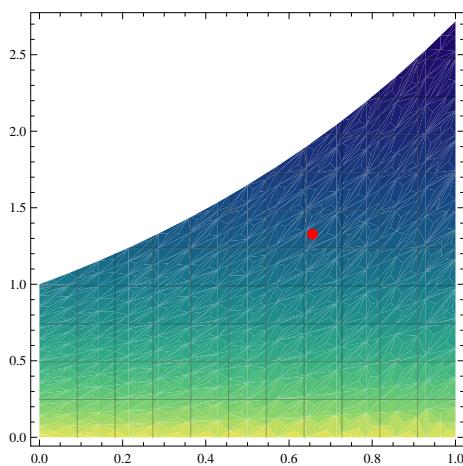
4. Finally compute \bar{x} and \bar{y} which are the exact coordinates of the center of mass.

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m} \iint_D x\rho(x, y) dA, \quad \bar{y} = \frac{M_x}{m} = \frac{1}{m} \iint_D y\rho(x, y) dA$$

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rho[x_, y_] := y
c[x_] := 0
d[x_] := E^x
a[y_] := 0
b[y_] := 1
mass = Integrate[rho[x, y], {x, c[y], d[x]}, {y, a[y], b[y]}]
xbar = 1/mass Integrate[x * rho[x, y], {x, c[y], d[x]}, {y, a[y], b[y]}]
ybar = 1/mass Integrate[y * rho[x, y], {x, c[y], d[x]}, {y, a[y], b[y]}]
Dens = DensityPlot[1 / (1 + rho[x, y]), {x, a[y], b[y]},
{y, c[x], d[x]}, Mesh -> 10, ColorFunction -> "BlueGreenYellow"];
COMDens = ListPlot[{{xbar, ybar}}, PlotStyle -> {PointSize[Large], Red}] ;
Show[Dens, COMDens]

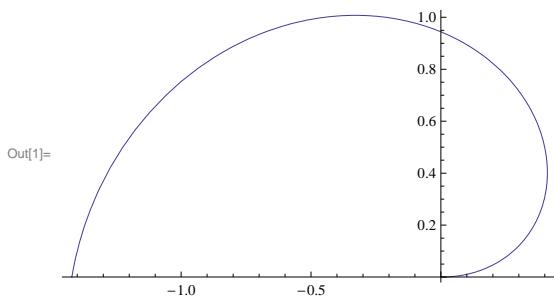
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Polar Coordinates

1. $r^2 = x^2 + y^2$
2. $x = r \cos \theta$
3. $y = r \sin \theta$
4. $\iint_R f(x, y) dA = \int_{\alpha}^{\beta} \int_a^b f(r \cos \theta, r \sin \theta) r dr d\theta$

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In[1]:= PolarPlot[Log[\theta + 1], {\theta, 0, \pi}]
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In[4]:= PolarPlot[Sin[10 \theta], {\theta, 0, \pi}]
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