# MA205 Lesson 37

Motion in Space

Monday, October 22, 2007

# Outline

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# Course Guide

## Objectives and Readings

Objectives

- Find a position function when acceleration, initial velocity, and initial position are all known.
- Understand the derivation of the physics equations of motion(the so-called kinematics equations).
- Use parametric equations to describe the motion of a projectile.

Read

- Stewart, Chapter 13, section 4, pages 870-878.

# Outline

# Course Guide

## Think About

- What does the size of a vector mean? How does this relate to acceleration and velocity?
- What assumptions are made to simplify the projectile motion problem? How would you include the things you assume do not effect projectile motion?

# Course Guide

## Mathematica Commands and Tasks You Need To Know

- The `Solve` command will be used a lot when solving projectile motion problems. It is designed to work with both normal and vector functions but you must be careful.
- For example if you know the position equation of a projectile is $\vec{r}(t) = \langle t^2 + 10, t^2 + 10, t^2 + 10 \rangle$.
- In order to solve for when the $x$ component is zero you have to use only the $x$ parametric equation using the following:
  `Solve[`$t^2 + 10$`==0,`$t$`]`
- If you wanted to solve for when the projectile was at the position $\langle 1, 1, 1 \rangle$ you would use:
  `Solve[r[t]=={1,1,1},t]`

# Motion in Space

- Similar to last class on Arc Length we start with a position vector
$$\vec{r}(t) = \langle t^2, e^t, te^t \rangle$$
- To get the velocity vector we simply take the derivative of the position vector
$$\vec{v}(t) = \vec{r}'(t) = \langle 2t, e^t, (1+t)e^t \rangle$$
- To get the speed of the particle we take the absolute value
$$|\vec{v}(t)| = \sqrt{4t^2 + e^{2t} + (1+t)^2 e^{2t}}$$

# Motion in Space

- Finally to find the particles acceleration we take the derivative of the velocity vector

$$\vec{a}(t) = \vec{v}'(t) = \langle 2, e^t, (2+t)e^t \rangle$$

# Outline

# Projectile Launching Part I

- Start the launcher at the origin like we are firing from the ground.
- Force due to gravity acts downward so we have $F = m\mathbf{a} = -mg\mathbf{j}$
- $g = |\mathbf{a}| \approx 9.8m/s^2$. or $g = |\mathbf{a}| \approx 32ft/s^2$. So $\mathbf{a} = -g\mathbf{j}$
- Since $\mathbf{v}'(t) = \mathbf{a}$, we have $\mathbf{v}(t) = -gt\mathbf{j} + C$
- $\mathbf{C} = \mathbf{v}(0) = \mathbf{v}_0$ So we get $\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}_0$
- Integrating again we get $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}_0 + \mathbf{D}$
- We now substitute in for $\mathbf{D}$ our initial position which if we start at the origin is zero

# Outline

# Projectile Launching Part II

- If we find the speed at time zero $|\mathbf{v}_0| = v_0$
- Then we get $\mathbf{v}_0 = v_0 \cos(\alpha) t\mathbf{i} + v_0 \sin(\alpha) t\mathbf{j}$
- So now we have $\mathbf{r}(t) = (v_0 \cos\alpha) t\mathbf{i} + \left[(v_0 \sin\alpha) t - \frac{1}{2} g t^2\right] \mathbf{j}$
- Our parametric equations of the mortar are

$$x = (v_0 \cos\alpha) t, \quad y = (v_0 \sin\alpha) t - \frac{1}{2} g t^2$$

# Outline

# Finding Initial Velocity

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$$x = (v_0 \cos \alpha)t + x_0, \quad y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2 + y_0$$

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- If we set these equations up and fill in all the known values like our angle $\alpha$, the angle our projectile is launched at, our initial positions, and our final positions, we get two equations and two unknowns.

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- If we set these equations up and fill in all the known values like our angle $\alpha$, the angle our projectile is launched at, our initial positions, and our final positions, we get two equations and two unknowns.
- We can now solve for one unknown like $v_0$, substitute it into the other equation and solve for $t$. Finally we substitute one last time and get a numerical answer for $v_0$.

# Outline

# Baseball Hit

- Simulate a hit of a baseball with your launcher
- Put a target out and measure the distance to it.
- Using the math and your known muzzle velocity adjust the angle to hit the target.
- BONUS POINTS FOR THE LEAST TRIES OR A FIRST TIME HIT

# Outline

# The Dreaded Thayer Board Work

- If a nine iron lofts a golf ball at an angle of $60^\circ$, how fast would you have to swing in order to hit the ball 30 feet?
- You are standing on a hilltop located at (10, 20, 300), all distances in meters. You want to launch mortars to the north. The gun tube is set at $70^\circ$ and the mortars fire with a velocity of 200 meters per second. With no wind, where will the projectiles land?

# Problem Solving Lab

Maybe Scorched Earth.

# Questions