MA205

Board Sheet - 22 October 2007, Motion in Space II, Lesson 37

These Steps Should Help You Compute Motion in Space

1) Start the particle at the origin like we are starting from the ground, or determine your original location in vector form.

2) Force due to gravity acts downward so we have $F = m\mathbf{a} = -mg\mathbf{j}$

3) $g = |\mathbf{a}| \approx 9.8m/s^2$. or $g = |\mathbf{a}| \approx 32ft/s^2$. So $\mathbf{a} = -g\mathbf{j}$

4) $\mathbf{v}'(t) = \mathbf{a}$, we have $\mathbf{v}(t) = -gt\mathbf{j} + C$

5) $\mathbf{C} = \mathbf{v}(0) = \mathbf{v}_0$ So we get $\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}_0$

6) Integrating again we get $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}_0 + \mathbf{D}$

7) We now substitute in for $\mathbf{D}$ our initial position which if we start at the origin is zero or a vector if we do not start at zero

8) If we find the speed at time zero $|\mathbf{v}_0| = v_0$ Then we get $\mathbf{v}_0 = v_0\cos(\alpha t)\mathbf{i} + v_0\sin(\alpha t)\mathbf{j}$

9) So now we have $\mathbf{r}(t) = (v_0\cos\alpha)t\mathbf{i} + \left[(v_0\sin\alpha)t - \frac{1}{2}gt^2\right]\mathbf{j}$

10) Our parametric equations of the mortar are

$$x = (v_0\cos\alpha)t + x_0, \quad y = (v_0\sin\alpha)t - \frac{1}{2}gt^2 + y_0$$

11) You now have two equations and two unknowns and can solve for both v_0 and t.

Now Try it on these Problems

If a nine iron lofts a golf ball at an angle of 60°, how fast would you have to swing in order to hit the ball 30 feet?

You are standing on a hilltop located at (10, 20, 300), all distances in meters. You want to launch mortars to the north. The gun tube is set at 70° and the mortars fire with a velocity of 200 meters per second. With no wind, where will the projectiles land?