MA205 Lesson 42

Slope Fields

Thursday, November 1, 2007

Outline

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Admin

1. CPRC Cadets need to let me know who you are. Your makeup WPR III will be 28 November 2007 during normal class period.
2. Engineer Experience

Admin

1. CPRC Cadets need to let me know who you are. Your makeup WPR III will be 28 November 2007 during normal class period.
2. Engineer Experience
3. Quiz - you will have 10 minutes

Who’s Birthday is it

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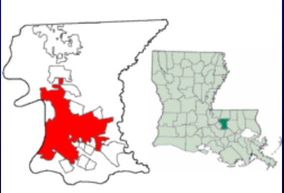


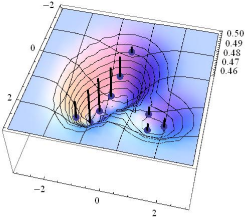
Who's Birthday is it

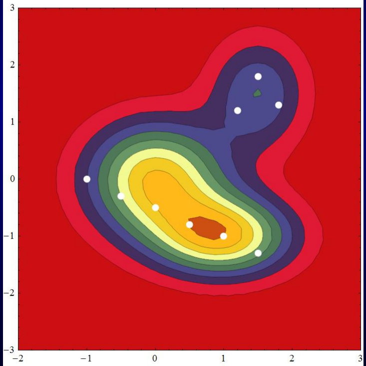
Alicia Chan

Solutions to Project

Project

```
(* MA205 Project 2 Option A *)
<< Units`
TeT[x_, y_] :=
 0.5 - 0.04 (E^-(x^2+y^2) + 0.5 E^-(2 (x-1.5)^2+(y-1.5)^2) + E^-((x-1)^2+2 (y+1)^2))
TeT[0, 0];
bouy3d = ListPointPlot3D[
   {{1.5, 1.8, TeT[1.5, 1.8] + .005}, {1.2, 1.2, TeT[1.2, 1.2] + .005},
    {1.8, 1.3, TeT[1.8, 1.3] + .005}, {-1, 0, TeT[-1, 0] + .005},
    {-0.5, -0.3, TeT[-0.5, -0.3] + .005}, {0, -0.5, TeT[0, -0.5] + .005},
    {0.5, -0.8, TeT[0.5, -0.8] + .005}, {1, -1, TeT[1, -1] + .005},
    {1.5, -1.3, TeT[1.5, -1.3] + .005}}, PlotStyle → {PointSize[Large]},
   Filling → 0.495, FillingStyle → {Thick, Orange},
   PlotRange → {{-2, 3}, {-3, 3}}];
lake3zmesh = Plot3D[TeT[x, y], {x, -2, 3}, {y, -3, 3},
   MeshFunctions -> {#3 &}, Mesh → 10];
lake3 = Plot3D[TeT[x, y], {x, -2, 3}, {y, -3, 3}, Mesh → 4];
Show[lake3, lake3zmesh, bouy3d]
lakec = ContourPlot[TeT[x, y], {x, -3, 3}, {y, -3, 3},
   Background → None, PlotRange -> {{-2.0, 3}, {-3, 3}},
   ContourShading → ColorData[35, "ColorList"]];
bouy = ListPlot[{{1.5, 1.8}, {1.2, 1.2}, {1.8, 1.3}, {-1, 0},
    {-0.5, -0.3}, {0, -0.5}, {0.5, -0.8}, {1, -1}, {1.5, -1.3}},
   PlotStyle → {White, PointSize[Large]}];
Show[{lakec, bouy}]
a = -2; b = 3; c = -3; d = 3;
DTeT[x_, y_] := If[TeT[x, y] ≤ 0.495, 0.495 - TeT[x, y], 0]
LakeV = NIntegrate[DTeT[x, y], {x, a, b}, {y, c, d}]
AveC = 3.57647626819842;
AveCC = Convert[AveC Micro Gram / Liter, MetricTon / Mile^3]
f[V_, A_] := 0.38 * V^-0.75 * A^0.25
f[LakeV, AveCC]
```





0.174301

$$\frac{14.9074\,\text{MetricTon}}{\text{Mile}^3}$$

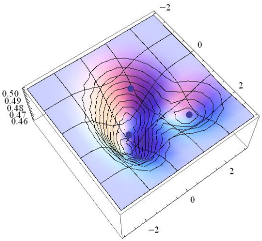
$$2.76796\left(\frac{\text{MetricTon}}{\text{Mile}^3}\right)^{0.25}$$

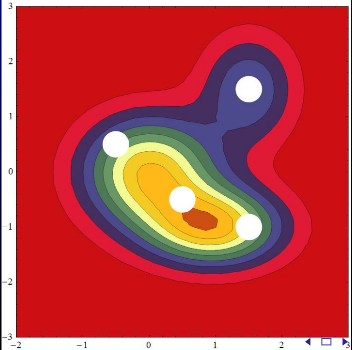
```
(* Option B *)
TeTx[x_, y_] = ∂x TeT[x, y];
TeTy[x_, y_] = ∂y TeT[x, y];
LakeBed[x_, y_] =
  If[TeT[x, y] < 0.495, √(1 + (TeTx[x, y])^2 + (TeTy[x, y])^2), 0];
LakeSA = NIntegrate[LakeBed[x, y], {x, -2, 3}, {y, -3, 3}]
RobotAve = 1.48585510802429;
RobotAveC = Convert[RobotAve Milli Gram / (Feet)^2,
  Kilo Gram / Mile^2]
Arsenic[SA_, Ava_] := 5.4 * SA^-0.75 * Ava^0.25;
Arsenic[LakeSA, RobotAveC]
Robot3d = ListPointPlot3D[
   {{1.5, 1.5, TeT[1.5, 1.5] + .005},
    {-0.5, 0.5, TeT[-0.5, 0.5] + .005},
    {0.5, -0.5, TeT[0.5, -0.5] + .005},
    {1.5, -1, TeT[1.5, -1] + .005}},
   PlotStyle → {PointSize[Large]},
   PlotRange → {{-2, 3}, {-3, 3}}];
Robot = ListPlot[{{1.5, 1.5}, {-0.5, 0.5}, {0.5, -0.5},
    {1.5, -1}}, PlotStyle → {White, PointSize[.08]}];
Show[lake3, lake3zmesh, Robot3d]
Show[{lakec, Robot}]
```

12.6184

$$\frac{41.4233\,\text{Gram Kilo}}{\text{Mile}^2}$$

$$2.04622\left(\frac{\text{Gram Kilo}}{\text{Mile}^2}\right)^{0.25}$$





Solutions to Homework

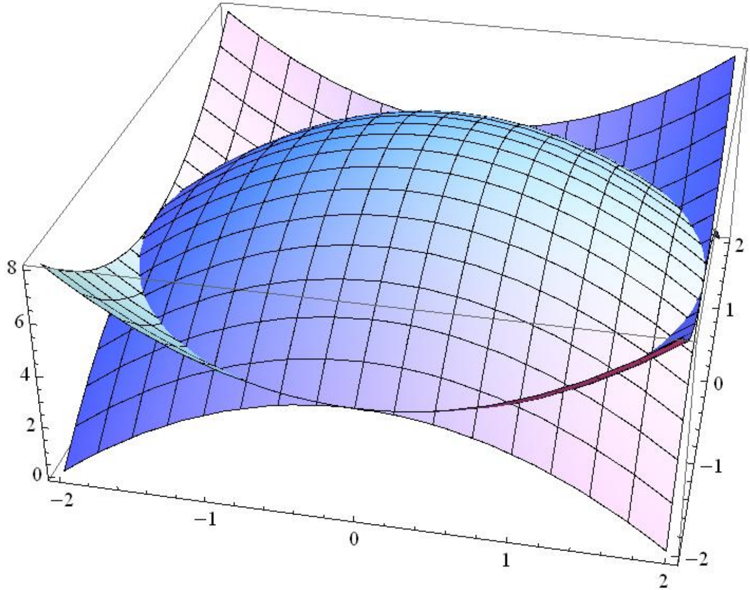
Homework

```
yeast1[x_, y_] := 8 - x^2 - y^2
yeast2[x_, y_] := x^2 + y^2
yeast1p = Plot3D[yeast1[x, y], {x, -2, 2}, {y, -2, 2}];
yeast2p = Plot3D[yeast2[x, y], {x, -2, 2}, {y, -2, 2}];
```

$$\int_{-2}^{2}\int_{-2}^{2}(\text{yeast1}[x, y] - \text{yeast2}[x, y])\, dy\, dx$$

```
Show[yeast2p, yeast1p]
```

$\frac{128}{3}$



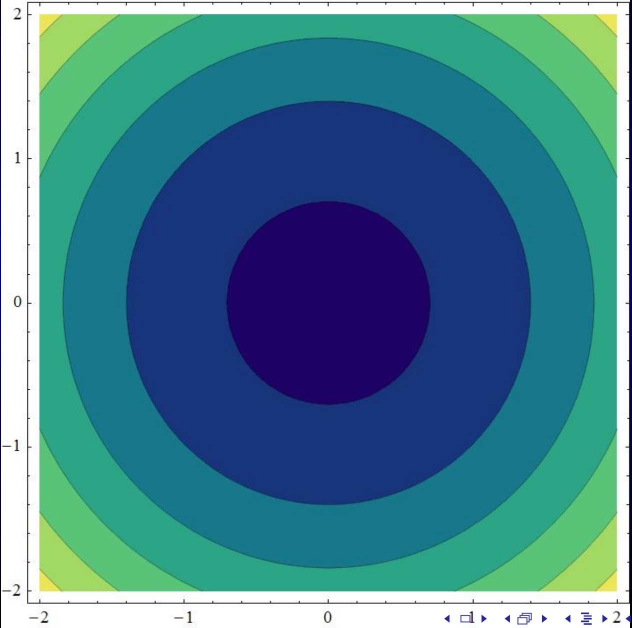
```
yeastc[x_, y_] := 1 000 000 * (300 - 3 x^2 - 3 y^2)
∫_{-2}^{2} ∫_{-2}^{2} yeastc[x, y] dx dy
1/16 ∫_{-2}^{2} ∫_{-2}^{2} yeastc[x, y] dx dy
ContourPlot[1 / yeastc[x, y], {x, -2, 2}, {y, -2, 2},
 Mesh → 10, ColorFunction → "BlueGreenYellow"]
```

$$\int_{-2}^{2}\int_{-2}^{2} \mathrm{yeastc}[x, y]\, dx\, dy$$

$$\frac{1}{16}\int_{-2}^{2}\int_{-2}^{2} \mathrm{yeastc}[x, y]\, dx\, dy$$

4 672 000 000

292 000 000



$$\text{yeastbsct}[x\_] := 2 - \frac{1}{2}x^2$$

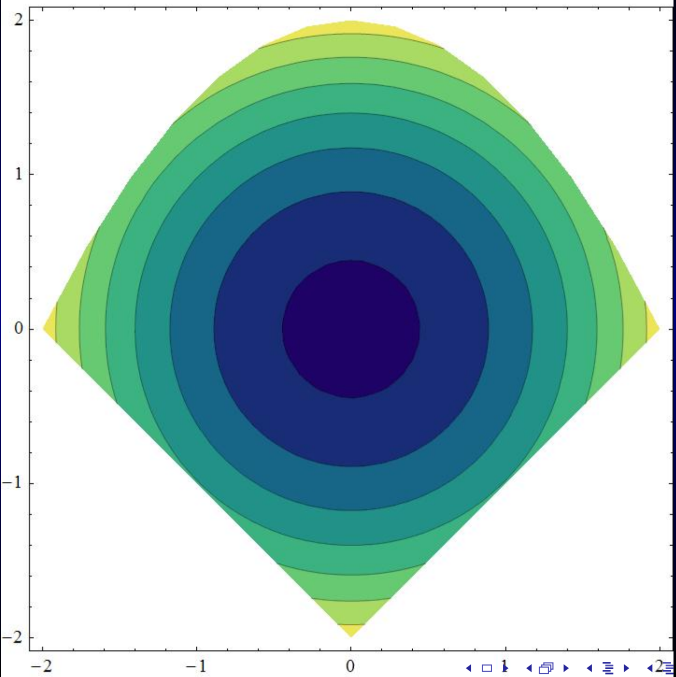
```
yeastbscb[x_] :=
  Piecewise[{{-2 - x, -2 ≤ x < 0}, {x - 2, 0 ≤ x ≤ 2}},
   {x, -2, 2}];
ytbs = Plot[yeastbsct[x], {x, -2, 2}, PlotRange → -2];
ybbs = Plot[yeastbscb[x], {x, -2, 2}];
Show[ytbs, ybbs];
```

$$\int_{-2}^{0}\int_{-2-x}^{2.-\frac{1}{2}x^2} \text{yeastc}[x, y]\,dy\,dx +$$

$$\int_{0}^{2}\int_{-2+x}^{2-\frac{1}{2}x^2} \text{yeastc}[x, y]\,dy\,dx$$

```
ContourPlot[1 / yeastc[x, y], {x, -2, 2},
 {y, yeastbsct[x], yeastbscb[x]}, Mesh → 10,
 ColorFunction → "BlueGreenYellow"]
```

2.75657×10^9



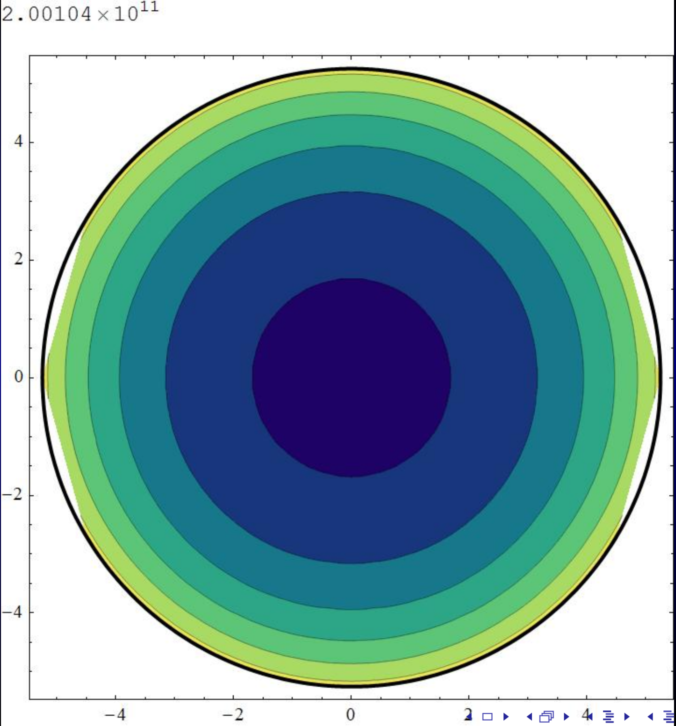
```
yeastcc[x_, y_] := 10 000 000 * (300 - 5 x^2 - 5 y^2)
yeastccp[θ_, r_] := yeastcc[r * Cos[θ], r * Sin[θ]]
```

$$\int_0^{2\pi}\int_0^{5.25} \text{yeastcc}[r * \text{Cos}[\theta], r * \text{Sin}[\theta]]\, r\, dr\, d\theta$$

$$\int_{-5.25}^{5.25}\int_{-\sqrt{5.25^2-x^2}}^{\sqrt{5.25^2-x^2}} \text{yeastcc}[x, y]\, dy\, dx$$

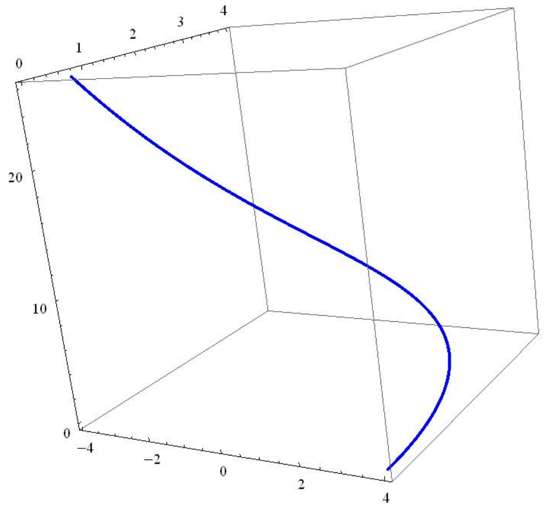
$$\int_0^{2\pi}\int_0^{5.25} \text{yeastccp}[\theta, r]\, r\, dr\, d\theta$$

```
yeastccplot = ContourPlot[1 / yeastcc[x, y],
    {x, -5.25, 5.25}, {y, -Sqrt[5.25^2 - x^2], Sqrt[5.25^2 - x^2]},
    Mesh → 10, ColorFunction → "BlueGreenYellow"];
carboy = PolarPlot[5.25, {θ, 0, 2 π},
    PlotStyle → {Thick, Black}];
Show[yeastccplot, carboy]
```


2.00104×10^{11}
4
2
0
−2
−4
−4
−2
0
2
4

```
yeastbagx[t_] := 4 Cos[t]
yeastbagy[t_] := 4 Sin[t]
yeastbagz[t_] := t^3
ParametricPlot3D[
 {yeastbagx[t], yeastbagy[t], yeastbagz[t]},
 {t, 0, 26^(1/3)}, BoxRatios → {1, 1, 1}, PlotPoints → 150,
 PlotRange → All, PlotStyle → {Thick, Blue}]
Solve[t^3 == 26, t] // N
Animate[ParametricPlot3D[
  {yeastbagx[t], yeastbagy[t], yeastbagz[t]},
  {t, 0, Z}, BoxRatios → {1, 1, 1}, PlotPoints → 150,
  PlotRange → {{-4, 4}, {-4, 4}, {0, 26}},
  PlotStyle → {Thick, Blue}], {Z, 0, 26^(1/3)}]
```

Out[144]=



Out[145]=

{{t → -1.48125 - 2.5656 i}, {t → 2.9625}, {t → -1.48125 + 2.5656 i}}

Outline

Course Guide

Objectives

Objectives

- Create a slope field for a given first order differential equation.
- Approximate a solution to a first order differential equation using a slope field.
- Discuss the long term behavior of a first order differential equation based on what the slope field looks like.

Course Guide

Readings

Read

- Stewart, Chapter 9, section 2, pages 592-598.

Course Guide

Think About

- Why use a graphical solution instead of a computed function?
- Can a graphical solution tell you more than a function can tell you?

Course Guide

Mathematica Commands and Tasks You Need To Know

- Follow directions in the course guide to install DETools exactly. This should be done already.

Outline

Intro to Differential Equations

Questions?

Outline

Slope Fields

1. Start up Mathematica
2. We will test Mathematica with a few problems
3. We will use the formula $y'(t) = y$ with initial conditions $y(0) = 0, y(0) = 2, y(0) = 4$ that we will trace on our screens.
4. Then we can input those curves in Mathematica to see how close we are.

More Slope Fields

1. Exponential Growth
2. Where $\frac{dP}{dt} = kP, k > 0$
3. Put this in DEPlot

Logistic Growth

1. Logistic Growth
2. Where $\frac{dP}{dt} = r(M-P)P, r > 0$
3. Put this in DEPlot

Test Problem

- Page 211 Course Guide

Did you work with Thayer Last Night?

- Do Mechanics Based Problem 1
- Then we will try PSP 4

Outline

Euler

Euler approximations

Questions