MA205 Lesson 49

OK! Enough Already! We haven't learned anything over the last month why add to it?

Wednesday, November 14, 2007

Outline

Course Guide

Last Class

Second Order Differential Equations

Characteristic Equation

Board Work

Lesson Link - Spring Mass

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Objectives

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- Given a general order, linear, homogenous differential equation, determine the characteristic or auxiliary equation.
- Determine a general solution to a given second order, linear, homogeneous differential equation.
- Find the particular solution to a second order, linear, homogenous initial-value problem.

Course Guide

Readings

Read

- Stewart, Chapter 17, section 1, pages 1141-1147.

Course Guide

Review

- Stewart, Chapter 9, section 1, pages 589, A model for the motion of a spring.

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Think About

- What is oscillation?
- What is the relationship between position velocity and acceleration?

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NEED TO KNOW MODELS

- Mixing Problems?
- Electrical Circuits?

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Second Order Differential Equations

1. Remember a couple of days ago?

Second Order Differential Equations

2. We did this weird board problem a where our solution was $y = e^{rt}$

Second Order Differential Equations

3. Did you figure out the shortcut? I know I didn't the first time!

Second Order Differential Equations

4. All you really need to remember is the quadratic equation!

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Characteristic Equation

1. It is called the characteristic equation or auxiliary equation

Characteristic Equation

2. Again our solution is $y = e^{rt}$ look at our equation!

Characteristic Equation

2. Again our solution is $y = e^{rt}$ look at our equation!
3. We use the same steps we learned in Lesson 41

Characteristic Equation

2. Again our solution is $y = e^{rt}$ look at our equation!

4. Take the derivative of our solution twice and substitute it into our differential equation!

Characteristic Equation

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4. Take the derivative of our solution twice and substitute it into our differential equation!
5. Now we get to the memorization part?

Characteristic Equation

1. We find that we get a Second Order, Homogeneous, Constant Coefficient, Linear Differential Equation! Great! What does that tell us?

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2. Well! Now we after some simplifying we get the quadratic equation.
3. We find the roots of the quadratic which tell us our solution.

Solution Based on Roots of the Quadratic Equation

1. The quadratic equation is when given $at^2 + bt + c = 0$ that our roots are given by
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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2. Now we get to more memorization
3. Use your book to see the proof, or memorize or put on your cheat sheet, the following

Solution Based on Roots of the Quadratic Equation

1. If the roots are real and different like $r = 1, -2$ then your solution is $y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$ where your first root is r_1 and your second root is r_2 So in our case we would have $y(t) = C_1 e^{1t} + C_2 e^{-2t}$

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2. If the roots are real and the same like $r = 3, 3$ then your solution is $y(t) = C_1 e^{rt} + C_2 t e^{rt}$ where your root is r. So in our case we would have $y(t) = C_1 e^{3t} + C_2 t e^{3t}$

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3. If the roots are imaginary $r = \alpha \pm \beta i$ like $r = 2 \pm 3i$ then your solution is $y(t) = e^{\alpha t}(C_1 \cos \beta t + C_2 \sin \beta t)$. So we would have $y(t) = e^{2t}(C_1 \cos 3t + C_2 \sin 3t)$

An Example

1. Problem 1 from the book page 1147 -

$$y'' - 6y' + 8y$$

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1. Problem 1 from the book page 1147 -

$$y'' - 6y' + 8y$$

2. After using the characteristic equation we get $r^2 - 6r + 8 = 0$
3. Factoring or using the quadratic we get $(r-2)(r-4)$ so our roots are $r = 2, 4$ which are two different real roots
4. So our solution looks like

$$y(t) = C_1 e^{2t} + C_2 e^{4t}$$

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An Initial Value Problem

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Mustang Suspension

Questions