MA205 Lesson 50

Spring-Mass Systems

Wednesday, 7 August, 2007 or Thursday, 15 November, 2007

Outline

1. Course Guide
   - Objectives and Readings
   - Think About and Mathmatica
2. Last Class Review - Analytic Solutions II: Second Order Differential Equations
3. Practical uses of Spring Mass
   - Forces acting on a mass spring system
4. Mass Spring
5. Mass Spring II and Problem Solving Lab VII

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Course Guide

Objectives

- Model initial value problems involving a spring mass system.
- Solve problems involving simple harmonic motion, free undamped motion, and damped motion.
- Understand critically damped, under damped, and over damped spring mass systems.
- Be able to describe the long term behavior of a spring mass system.

Read

- Stewart, Chapter 17, section 3, pages 1155-1159

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Course Guide

Think About

- What is the form for any linear second order differential equation both homogeneous and non homogeneous?
- What is the form of any imaginary number?

Mathematica Commands and Tasks You Need To Know

- DSolve

Last Class Review - Analytical Solutions

Auxiliary Equation!
Questions?

Practical uses of Spring Mass

- Spring hanging from ceiling or rod with mass attached.
- Spring used in car suspension. Like in a Mustang.

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- Spring Force or the spring's stiffness.
- Damping Force or the force slowing down the movement of the spring.
- Gravity's Force: the force of gravity pulling on the mass and spring.
- Driving Force: a force causing motion in the system.

Mass Spring - Second-Order Differential Equation

- $$my'' + \delta y' + ky = F(t)$$

or

- $$my'' + cy' + ky = F(t)$$

Variables of Mass Spring Equation

- m = mass
  - English Units : $m = w/g \Rightarrow m = \frac{lb}{32\frac{ft}{sec}} = slug$
  - Metric Units : $m = kg$
- $\delta = damping$ or $c = damping$: Any damping force acts in a direction opposite to the velocity y'.
- k = spring constant
  - English Units : $k = \frac{mg}{ft} \Rightarrow k = \frac{lb \cdot 32\frac{ft}{sec}}{ft} \Rightarrow k = \frac{lb}{ft}$
  - Metric Units : $k = \frac{N}{m}$
- $F(t) =$ any driving force

Initial Conditions

- $y(0) =$ location of mass at time zero.
- $y'(0) =$ velocity of mass at time zero.

Board Work on Mass Spring Systems

- Team 1 : A mass of 0.6 kg is attached to the free end of a suspended spring, stretching the spring 0.4 m beyond its natural length. The mass is then pulled down and additional 1 meter and released from rest.
  - a.) Use Newton's second law to model the position of the mass.
  - b.) Use Mathmatica DSolve and Plot to evaluate the system.
  - c.) Access the URL Spring Mass Cartoon to compare your answer.
  - d.) Add dampening to see how to reduce the amount of oscillations. Check your dampened system in both Mathmatica and the above website.

Board Work on Mass Spring Systems

- Team 2 : An object weighing 4 lbs is attached to the free end of a spring, stretching the spring 6 inches. The spring-mass system is in a medium which creates a damping force numerically equal to the object's velocity.
  - a.) If the object is released from a location 2 feet above it's equilibrium position with a downward velocity of 3 ft/sec, use Newton's secon law to model the position of the object.
  - b.) Access the URL Spring Mass Cartoon to evaluate the system (you will have to convert units to metric).
  - c.) Use Mathmatica DSolve and Plot to evaluate the system if a motor applies an external force of $10\cos(8t)$.
  - d.) Vary the dampening to evaluate the effect on oscillations (exclude the motor force). Check your dampened system in both Mathmatica and the above website.

Board Work on Mass Spring Systems

- Team 3 : Major Bowman's 1965 Mustang weighs 3000 lbs which is approximately 2000 lbs on the front wheels and 1000 lbs on the rear wheals. The front springs have a stiffness of 260 lbs/in.
  - a.) Model the the position of one of the front wheels of the car if the car drove across a 6 inch bump.
  - b.) Access the URL Spring Mass Cartoon to evaluate the system you just modeled (you will have to convert units to metric) assume there is no dampening at this time.
  - c.) Use Mathmatica DSolve and Plot to evaluate the system and compare it to what you found on the web site.
  - d.) Add dampening (i.e. a shock absorber) to evaluate the effect on oscillations. Check your dampened system in both Mathmatica and the above website. See if you can find the dampening needed to limit oscillations to two

Mass Spring II and Problem Solving Lab VII

Questions Regarding Mass Spring Systems

Questions